International Environmental Agreements with Uncertainty, Learning and Risk Aversion

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Abstract

Uncertainty and learning play an important role for the formation of international environmental agreements (IEAs). For instance, scientific uncertainty about climate damages and technological abatement and mitigations options is still large despite ongoing research. It has been shown that in the strategic context of voluntary participation but strong free-rider incentives, learning may have a negative impact on the success of IEAs. This paper extends the model of Kolstad (2007) and Kolstad and Ulph (2008) by considering risk aversion. This seems suggestive as uncertainties in climate change are highly correlated and hence pooling risks may be limited. It is shown that the negative conclusion with respect to the role of learning derived for risk neutrality has to be qualified.

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1. Introduction

Environmental issues such as climate change pose four key challenges for economic analysis: (i) the process of climate change is effectively *irreversible*; (ii) there are considerable *uncertainties* about the likely future costs of both abatement, but more especially environmental damages; (iii) our understanding of these uncertainties changes over time as a result of *learning* more about climate science, possible technological responses and behavioural responses by households, firms and governments; (iv) the problem is global, but since there is no single global agency to tackle climate change, policies need to be negotiated through *international environmental agreements.*

The first three issues have been studied quite extensively[1] in the context of a single global government where the key issue is should the possibility of future learning in a problem with uncertainty and irreversibility lead to more or less current action. The *precautionary principle* argues for more current action, but the theoretical and empirical analysis is more ambiguous. There has also been an extensive literature[2], both theoretical and empirical, on the fourth issue, but mainly in the context of complete certainty about the net benefits of tackling climate change. The conclusions have been rather pessimistic, in the sense that while there are substantial benefits to all countries collaborating to tackle climate change, relative to countries acting non-cooperatively, if countries decide independently whether to join an international environment agreement (IEA) the gains from such agreements are small.

More recently these two strands of literature have begun to be integrated. Ulph and Ulph (1996), Ulph and Maddison (1997) compare the fully cooperative and the non-cooperative when countries face uncertainty about damage costs. They show that the value of learning about damage costs may be negative when countries act non-cooperatively and damage costs are negatively correlated across countries. Na and Shin (1998), Ulph (2004), Kolstad (2007), Kolstad and Ulph (2008, 2009) have considered

1 See, for example, Arrow and Fisher (1974), Epstein (1980), Kolstad (1996a,b), Gollier, Julien and Treich (2000), Ulph and Ulph (1997) as well as Narain, Fisher and Hanemann (2007).

2 Classic papers are Barrett (1994), Carraro and Siniscalco (1993) while for instance Finus (2001, 2003) and Barrett (2003) provide surveys of the literature.

how the prospect of future resolution of uncertainty affects the incentives for countries to join an IEA. Again the results have been rather pessimistic.

Kolstad and Ulph (2008) consider a model where countries face common uncertainty about the level of environmental damage costs.[3] They consider three types of learning: with *full learning* uncertainty about damage costs is resolved before countries decide whether or not to join an IEA; with *partial learning*, uncertainty is resolved after countries decide whether or not to join an IEA, but before they choose their emissions levels; with *no learning* uncertainty is not resolved until countries have decided whether or not to join an IEA and set their emission levels. Kolstad and Ulph (2008) show that the prospect of learning, either full or partial, generally reduces the expected welfare in stable IEAs.

All these models have assumed that countries are risk neutral. However, in the climate context, risks are highly correlated and hence possibilities for risk-sharing are limited so that the assumption of risk aversion may be quite relevant. Endres and Ohl (2003) show in a simple prisoners' dilemma that risk aversion increases the prospects for cooperation once it reaches a certain threshold. Bramoullé and Treich (2006) integrate risk aversion in a global emission model that compares the non-cooperative with the fully cooperative solution. They show that equilibrium emissions are lower with uncertainty and this difference increase with the degree of risk aversion as part of a hedging strategy but the effect on global welfare is ambiguous. Boucher and Bramoullé (2009) use the model of Kolstad and Ulph (2008) to analyze the effect of risk aversion on coalition formation as we do in this paper. In contrast to us, they use the more sophisticated expected utility approach whereas we employ the mean standard deviation approach. However, they do not consider the case of partial learning that appears to be particular relevant in actual treaty-making and displays interesting strategic features. Moreover, they do not consider the benchmark case of full learning.

[3] By common uncertainty we mean that each country faces the same *ex ante* distribution of possible damage costs, and when uncertainty is fully resolved they face the same *ex post* level of damage costs, i.e. the risks they face are fully correlated across countries. Kolstad and Ulph (2009) extend this model to consider the case where the risks each country faces are uncorrelated. Uncorrelated uncertainty is also considered in a slightly different model in Finus and Pintassilgo (2009) and empirically investigated in a climate model with twelve world regions in Dellink et al. (2008).

The paper proceeds as follows. In section 2, we set out the theoretical model. We first summarise the benchmark cases: certainty and then uncertainty considering three models of learning, assuming risk neutrality. Then, we move on to derive the case with uncertainty and learning, considering risk aversion. In section 3, we present some numerical simulations as not all results can be derived analytically in section 2. Section 4 summarises our main results.

2. The Model

2.1 No Uncertainty

There are N identical countries, indexed $I = 1,...,N$. Each country produces emissions x_i which we assume can take one of two values: $x_1 = 1$ (pollute) or $x_i = 0$ (abate). Aggregate emissions are denoted by $X = \sum_{i=1}^{N} x_i$. Aggregate emissions cause global environmental damages. The cost of environmental damages per unit of global emissions is γ and the benefit per unit of individual emissions is normalized to 1. (Thus, γ essentially measures the cost-benefit ratio.) The payoff to country i is given by

$$\Pi_i(x_i ,X) \equiv x_i - \gamma X \quad . \tag{1}$$

In order to make this model interesting, we require that the individual benefit exceeds the individual unit damage cost from pollution, i.e. $1 > \gamma$ (hence countries pollute in the Nash equilibrium) but not the global unit damage cost, i.e. $1 < \gamma N$ (hence countries abate in the social optimum), which together implies:

Assumption 1: $\quad 1/N < \gamma < 1$.

In order to study IEA formation, we shall use the simple model of Barrett (1994), borrowed from the literature on cartel formation (d'Aspremont et al. 1983). This uses a two-stage game approach. In the second stage, the *emission game*, for any arbitrary number of IEA members n , $1 \le n \le N$, the members of the IEA (which we shall denote by the symbol *c*, representing *coalition* country) and the remaining countries (which we denote by the symbol *f*, representing *fringe* country) set their emission levels as the outcome of Nash game between the coalition and the fringe countries. That is, the coalition members together maximize the aggregate payoff to their coalition whereas as fringe countries maximize their own payoff. As we assumed $1>\gamma$, $x_i^f=1$; coalition members will chose $x_i^c=0$ provided $1\le\gamma n$, and so $\Pi_i^f(n)=1-\gamma(N-n)$ and $\Pi_i^c(n)=-\gamma(N-n)$[4]; however if $1>\gamma n$, then coalition members will choose $x_i^c=1$ and so $\Pi_i^c(n)=\Pi_i^f(n)=1-\gamma N$.

Knowing the payoffs to coalition and fringe countries for any arbitrary number of IEA members, we then determine the equilibrium in the first stage, the *membership game*, again as a Nash equilibrium: no coalition country could become better off by defecting from the coalition, and no fringe country could be made better off by joining the coalition:

Internal stability: $\forall\, i\in C: \Pi_i^c(n)\ge\Pi_i^f(n-1)$ (2)

External stability: $\forall\, i\notin C: \Pi_i^f(n)\ge\Pi_i^c(n+1)$. (3)

Consider a coalition with n members that abates because $1\le\gamma n$. Now if one member leaves the coalition and it still paid the remaining $n-1$ to abate, i.e. $1\le\gamma(n-1)$, internal stability $\Pi_i^c(n)\ge\Pi_i^f(n-1)$ would require $0-\gamma(N-n)\ge 1-\gamma(N-n+1)$ $\Leftrightarrow \gamma\ge 1$ which violates Assumption 1. Hence, we require that it does not pay the remaining countries to abate, i.e. $1>\gamma(n-1)$, and then internal stability requires $0-\gamma(N-n)\ge 1-\gamma N$ which implies that $n\ge 1/\gamma$. Thus, the internally stable coalition n^* is the smallest integer $I(\gamma)$ no less than $1/\gamma$ which can be easily checked is also externally stable and hence stable. It is straightforward to show[5] that $I(\gamma)$ is a decreasing and convex function of γ.

Since *ex ante* all countries are identical, there is no explicit process for determining which countries get selected as IEA members and which as fringe countries. We shall assume, following Kolstad and Ulph (2008) and Rubio and Ulph (2007) that there is an implicit random process for determining which countries become IEA members. Thus,

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4 It is now evident why we need Assumption 1: it avoids trivial outcomes where all countries either abate or pollute no matter whether they are coalition members.

5 Ignoring the integer nature of $I(\gamma)$ *so that* $I(\gamma)=1/\gamma$, *then* $I'(\gamma)=-\gamma^{-2}<0$ and $I''(\gamma)=2\gamma^{-3}>0$.

we define the average or expected payoff by $\omega(\gamma) \equiv \left(n^* / N\right) \Pi_i^c + \left((N - n^*)/N\right) \Pi_i^f$. Then, it can be shown that $\omega(\gamma)$ is a decreasing and concave function in γ.[6]

Thus, this simple model provides a relationship between the unit damage cost γ and the equilibrium number of coalition members. The equilibrium is a knife-edge equilibrium with n^* countries forming the coalition which de facto dissolves once a member leaves the coalition as no country would abate anymore.

2.2 Uncertainty

2.2.1 Introduction

Now assume that the unit damage cost of global emissions is uncertain; it is assumed to be the same for all countries, both *ex ante* and *ex post*. We denote the value by γ_s in state of the world *s* and hence (1) becomes:

$$\Pi_{i,s}(x_i, X) \equiv x_i - \gamma_s X \quad . \tag{4}$$

For simplicity, we assume that γ_s can take one of two values: *low* damage costs, γ_l, with probability p, and *high* damage costs, γ_h with probability $(1-p)$ where $\gamma_l < \gamma_h$ and $0 \le p \le 1$. We denote by $\overline{\gamma} \equiv p\gamma_l + (1-p)\gamma_h$ the expected value of unit damage costs, and by $\sigma(\gamma_s) \equiv (\gamma_h - \gamma_l)\sqrt{p(1-p)}$ the standard deviation of unit damage costs.

To assess how countries evaluate payoffs across states of the world, we assume that each country's attitudes to risk can be represented by a *mean-standard deviation* (MS) welfare function which is the same for all countries:

$$V_i(x_i, X) \equiv E\left[\Pi_i(x_i, X)\right] - \alpha\sigma\left[\Pi_i(x_i, X)\right] \tag{5}$$

where $\alpha \ge 0$ is the coefficient of risk aversion; i.e. the payoff to a country is the expected payoff minus the standard deviation of payoffs weighted by the factor α. $\alpha = 0$ corresponds to risk neutrality. The MS decision criterion, introduced by Markowitz (1952) and Tobin (1958), has been compared with what is regarded as a more general *expected utility* (EU) criterion for decision-making under uncertainty, introduced by von Neumann and Morgenstern (1944). There have been a large number

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6 Again ignoring the integer nature of $I(\gamma)$, $\omega(\gamma) = 2 - N\gamma - 1/(N\gamma)$, $\omega'(\gamma) = -N(1 - 1/(N\gamma)^2) < 0$ and $\omega''(\gamma) = -2/(N\gamma^3) < 0$.

of studies comparing the relative advantages of each approach and asking under which conditions are they consistent – i.e. they produce the same results. In a path-breaking article Meyers (1987) showed that under a given parameter restriction (LS – location and scale) the two approaches are consistent, and that the LS restriction holds in a wide variety of economic models. The MS approach has also been widely used in the empirical and applied literature because of its simplicity (Saha, 1997).

For later purposes, it will be useful to define:

$$\hat{\gamma} \equiv \overline{\gamma} + \alpha\sigma(\gamma_s) \geq \overline{\gamma} \tag{6}$$

as the 'risk-adjusted expected unit damage cost'.

While *ex ante* countries face uncertainty about the true value of unit damage costs, we want to allow for the possibility that countries may learn information during the course of the game which changes the risks they face. We shall follow Kolstad and Ulph (2008) in considering three very simple models of *learning*. With *No Learning (NL)* countries make their decisions about membership and emissions with uncertainty about the true value of unit damage costs. With *Full Learning (FL)* countries learn the true value of unit damage costs before they have to make their decisions on either membership or emissions. With *Partial Learning (PL)* countries learn the true value of damage costs after they have made their membership decisions but before they make their emission decisions. Thus, in this simple analysis, learning takes the form of revealing perfect information.

Like in the model without uncertainty, we have to introduce some parameter restrictions. Moreover, the equilibrium size of the coalition can be related to unit damage costs. We then define $n_h \equiv I(\gamma_h)$, $\overline{n} \equiv I(\overline{\gamma})$, $n_l \equiv I(\gamma_l)$ and $\hat{n} \equiv I(\hat{\gamma})$. It will turn out that the relevant stable IEAs will take one of these values. For sensible results, we make the following assumption.

Assumption 2: (i) $1/N < \gamma_l < \overline{\gamma} < \gamma_h < 1;$ (ii) $1/N < \overline{\gamma} \leq \hat{\gamma} < 1 \Rightarrow \alpha < (1-\overline{\gamma})/\sigma(\gamma_s)$
(iii) $n_h << \overline{n} << n_l$.

Assumptions 2(i) and (ii) are essentially Assumption 1 in the context of uncertainty. Note that Assumption 2(ii) imposes an upper limit on the degree of risk aversion which we can allow for, though as we shall see this does not have much effect on our results. The last assumption is not essential for our results, but is designed to ensure that uncertainty about unit damage costs is sufficiently great that it results in distinctly different sizes of possible stable IEAs – i.e. uncertainty matters for the size of IEA.

Note that now with uncertainty the expected welfare from an *ex ante* perspective is $\omega(\gamma) \equiv (\tilde{n}/N)V_i^c + ((N-\tilde{n})/N)V_i^f$ where $\tilde{n}$ may take on one of the values mentioned above (i.e. n_h, $\bar{n}$, n_l or $\hat{n}$). Thus, there are now two sources of uncertainty and risk in this model: the implicit uncertainty of not knowing whether one will be a coalition or a fringe country and the explicit uncertainty of not knowing what unit damage costs might be. This is especially important when we are modelling risk aversion and comparing outcomes from different models of learning, to which we now turn.

2.2.2 Risk Neutrality

In this subsection, we briefly summarise the results from Kolstad and Ulph (2008) for the case of risk neutrality ($\alpha = 0$) to provide a benchmark for comparison with the new results of this paper where countries are risk averse.

Result 1: No Learning with Risk Neutrality

In the emission game, fringe countries always pollute; coalition members abate if $n \geq \bar{n} = I(\bar{\gamma})$ *and pollute otherwise. In the membership game, the unique stable IEA has* $n_{NL,RN} = \bar{n}$ *members which abate, and* $N - \bar{n}$ *fringe countries which pollute. The expected welfare of a coalition member is* $V^c_{NL,RN} = -\bar{\gamma}(N-\bar{n})$ *while for a fringe country the expected payoff is* $V^f_{NL,RN} = 1 - \bar{\gamma}(N-\bar{n})$*. Taking account of the risk of being a coalition member or a fringe country, expected payoff per country with No Learning and Risk Neutrality is* $V_{NL,RN} = \omega(\bar{\gamma}) = (N-\bar{n})(1/N - \bar{\gamma})$*.*

This result displays *certainty equivalence* in the sense that the outcome is the same as would be obtained if countries faced unit damage costs $\bar{\gamma}$ with certainty.

Result 2: Full Learning with Risk Neutrality

If state $s = l, h$ *has been revealed at the outset, then in the emission game fringe members always pollute, and coalition members abate if* $n \geq n_s = I(\gamma_s)$. *In the membership game, the stable IEA has* n_s *members; the payoff to a coalition member is* $V^c_{FL,RN}(s) = -\gamma_s(N - n_s)$ *and to a fringe country is* $V^f_{FL,RN}(s) = 1 - \gamma_s(N - n_s)$. *Taking account of the risk of being a coalition member or a fringe country in state* s, *the expected payoff per country is* $V_{FL,RN}(s) = \omega(\gamma_s) = (N - n_s)(1/N - \gamma_s)$. *The expected size of an IEA with Full Learning and Risk Neutrality is* $n_{FL,RN} = pn_l + (1-p)n_h$ *and the expected payoff per country, taking account of the risk of high or low damages, is:*
$V_{FL,RN} = p\omega(\gamma_l) + (1-p)\omega(\gamma_h) = p(N - n_l)(1/N - \gamma_l) + (1-p)(N - n_h)(1/N - \gamma_h)$.

With Full Learning, the outcome in each state s in terms of the size of a stable IEA, payoffs to coalition and fringe countries and hence expected payoff to a country is obviously just the same as an IEA game where the level of damage costs γ_s is known with certainty, and we then take expectations across the two states of the world to obtain expected size of an IEA and expected payoff per country.

Before discussing the outcome with Partial Learning, it will be useful to introduce the following notation: let $\varepsilon \equiv n_l - 1/\gamma_l$ which is the difference between smallest integer not less than $1/\gamma_l$ and $1/\gamma_l$ and hence $0 \leq \varepsilon < 1$ and define:

$$\tilde{p} \equiv \frac{1 - \gamma_h}{1 - \gamma_h + \varepsilon\gamma_l}. \tag{7}$$

Result 3: Partial Learning with Risk Neutrality

In the emissions game, fringe countries always pollute; coalition members abate in both states of the world if $n \geq n_l$, *pollute in both states of the world if* $n < n_h$, *and pollute in state* $s = l$ *and abate in state* $s = h$ *if* $n_h \leq n < n_l$. *In the membership game, the IEA with* $n_{PL,RN} = n_h$ *members is always stable; all countries pollute in the low damage cost state, while coalition members abate in the high damage cost state. If* $p > \tilde{p}$, *then there is a second stable IEA with* $n_{PL,RN} = n_l$ *members in which fringe countries always pollute and IEA members always abate.*

In the first equilibrium with $n_{PL,RN} = n_h$ *members, the expected payoff to a coalition member is* $V^c_{PL,RN}(n_h) = p(1-N\gamma_l) - (1-p)\gamma_h(N-n_h)$ *while the expected payoff to a fringe country is* $V^f_{PL,RN}(n_h) = p(1-N\gamma_l) + (1-p)[1-\gamma_h(N-n_h)]$. *Taking account of the risk of being a coalition or fringe country, the expected payoff per country is given by* $V_{PL,RN}(n_h) = (1-N\bar{\gamma}) + (1-p)n_h(\gamma_h - 1/N)$.
In the second equilibrium with $n_{PL,RN} = n_l$ *members, the expected payoff to a coalition member is* $V^c_{PL,RN}(n_l) = -\bar{\gamma}(N-n_l)$ *while the expected payoff to a fringe country is* $V^f_{PL,RN}(n_l) = 1-\bar{\gamma}(N-n_l)$. *Taking account of the risk of being a coalition or fringe country, the expected payoff per country is given by* $V_{PL,RN}(n_l) = (N-n_l)(1/N-\bar{\gamma})$.

Comparing the outcomes in terms of expected size of the IEA and expected payoff per country across the three models of learning with Risk Neutrality, we find the following:

<u>Result 4: Comparison of Outcomes</u>

(i) $n_{PL,RN} = n_h \le n_{NL,RN} = \bar{n} \le n_{FL,RN} = (pn_l + (1-p)n_h) \le n_{PL,RN} = n_l$

(ii) $V_{PL,RN}(n_h) \le V_{FL,RN} \le V_{NL,RN} \le V_{PL,RN}(n_l)$.

With respect to the ranking of the expected size of the IEA, the inequality between No Learning and Full Learning follows from the convexity of $I(\gamma)$ (see footnote 5) and the other relations from Assumption 2 and $0 \le p \le 1$. In terms of the expected payoff per country, the inequality between Full Learning and No Learning follows from the concavity of $\omega(\gamma)$ (see footnote 6); the inequality between No Learning and Partial Learning with the high membership outcome, n_l, follows from the fact that $n_l > \bar{n}$ and $\bar{\gamma} > 1/N$ by Assumption 2; the inequality between Partial Learning and the low membership outcome, n_h, and Full Learning is derived in Kolstad and Ulph (2008).

In summary, with Risk Neutrality, if we view the parameter values $p > \tilde{p}$ generating $n_{PL,RN} = n_l$ as being rather uninteresting (i.e. we are interested in cases where the risk of high damage costs are quite significant), then we conclude that learning, whether full or partial, reduces welfare.

2.2.3 Risk Aversion

We now analyse how the results for Risk Neutrality carry over to Risk Aversion where the coefficient of risk aversion α in (5) is strictly positive. We introduce some additional notation. Recall that expected risk-adjusted unit damage costs as defined in (6) is $\hat{\gamma}$ and let the corresponding stable IEA have $\hat{n} = I(\hat{\gamma})$ members, such that each country faces a probability $\hat{n}/N$ of being an IEA member with welfare $-\hat{\gamma}(N-\hat{n})$, and a probability $1-(\hat{n}/N)$ of being a fringe country with welfare $1-\hat{\gamma}(N-\hat{n})$. Then the *ex ante* expected risk-adjusted payoff per country is given by $V(\hat{\gamma}) = E[\omega(\hat{\gamma})] - \alpha\sigma[\omega(\hat{\gamma})]$ or:

$$\rho(\hat{\gamma}) = \omega(\hat{\gamma}) - \alpha\delta(\hat{\gamma}) \text{ where } \delta(\hat{\gamma}) = \sqrt{(1/N\hat{\gamma})[1-(1/N\hat{\gamma})]} \quad . \tag{8}$$

The first term is just expected welfare. The term $\sigma[\omega(\hat{\gamma})]$ measures the standard deviation of the risk of being a signatory or non-signatory; the difference in payoffs is just 1; so the term $\delta(\hat{\gamma})$ just measures the square root of the probability of being a coalition member multiplied by the probability of being a fringe country where we use the approximation $\hat{n} = I(\hat{\gamma}) \cong 1/\hat{\gamma}$. Now we know that $\omega'(\gamma) < 0, \omega''(\gamma) < 0$ (see footnote 6). It is straightforward to show[7] that $\delta'(\hat{\gamma}) \geq 0 \Leftrightarrow \hat{\gamma} \leq 2/N$, i.e. $\hat{n} \geq N/2$. The rationale is that $\delta(\hat{\gamma})$ is zero when all countries are either signatories ($\hat{\gamma} = 1/N$) or non-signatories ($\hat{\gamma} = 1$) and reaches its maximum value (0.5) when half the countries are signatories and half are non-signatories. Hence, $\rho'(\hat{\gamma}) < 0$ if $\hat{\gamma} \leq 2/N$, i.e. $\hat{n} \geq N/2$, but otherwise the sign is ambiguous. Since from Assumption 2, $1/N \leq \hat{\gamma} < 1$, $\hat{\gamma} > 2/N$ cannot be regarded as unlikely and hence the effect of risk on per country payoff is non-trivial.

No Learning

In the emissions and membership game the outcomes are very similar to the case with Risk Neutrality, except that countries base their decisions on the 'risk-adjusted' expected unit damage cost $\hat{\gamma} = \overline{\gamma} + \alpha\sigma(\gamma_s)$ rather than on expected unit damage cost $\overline{\gamma}$.

[7] Let $q(\hat{\gamma})$ be the probability of a country being a signatory where $q(\hat{\gamma}) = 1/N\hat{\gamma}$; then $\delta(\hat{\gamma}) = \sqrt{q(\hat{\gamma})(1-q(\hat{\gamma}))}$, $d\delta/d\hat{\gamma} = (d\delta/dq)(dq/d\hat{\gamma})$, $dq/d\hat{\gamma} < 0$ and $d\delta/dq \geq 0 \Leftrightarrow q \leq 1/2$.

Result 5: No Learning with Risk Aversion

In the emission game, fringe countries always pollute, while coalition members abate if $n \geq \hat{n}$ *and pollute otherwise. In the membership game, the unique stable IEA has* $n_{NL,RA} = \hat{n}$ *members abating and* $(N - \hat{n})$ *fringe countries polluting. The expected payoff to a coalition member is* $V^{c}_{NL,RA} = -\hat{\gamma}(N - \hat{n})$ *and to a fringe country it is* $V^{f}_{NL,RA} = 1 - \hat{\gamma}(N - \hat{n})$. *Taking account of the risk of being a coalition member or a fringe country, expected payoff per country with No Learning and Risk Aversion is given by* $V_{NL,RA} = \rho(\hat{\gamma})$.

Comparing the results for risk aversion with those with risk neutrality, we have: $n_{NL,RA} = I(\hat{\gamma}) \leq I(\overline{\gamma}) = n_{NL,RA}$ and $V_{NL,RA} = \omega(\hat{\gamma}) - \alpha\delta(\hat{\gamma}) < \omega(\hat{\gamma}) < \omega(\overline{\gamma}) = V_{NL,RA}$. Thus, risk aversion leads to a smaller IEA and lower per country welfare than risk neutrality.

More generally, we are interested in the comparative statics of a change in the risk of high or low damages and in risk aversion. We have:

$$\frac{\partial n_{NL,RA}}{\partial x} = I'(\hat{\gamma}) \frac{\partial \hat{\gamma}}{\partial x} < 0 \quad \text{where} \quad x = \alpha, \sigma(\gamma_s) \tag{9}$$

since $I'(\hat{\gamma}) < 0$ and $\partial\hat{\gamma}/\partial x > 0$. Thus, increasing either risk aversion (i.e. α) or risk (i.e. standard deviation) reduces the size of an IEA.

$$\frac{\partial V_{NL,RA}}{\partial \alpha} = \sigma(\gamma_s)[\omega'(\hat{\gamma}) - \alpha\delta'(\hat{\gamma})] - \delta(\hat{\gamma}) \quad \text{and} \quad \frac{\partial V_{NL,RA}}{\partial \sigma(\gamma_s)} = \alpha[\omega'(\hat{\gamma}) - \alpha\delta'(\hat{\gamma})] \tag{10}$$

Both of these derivatives are negative if $\hat{\gamma} \leq 2/N$ (i.e. $\delta'(\hat{\gamma}) \geq 0$), otherwise they are ambiguous in sign. Thus, it is possible that an increase in risk aversion or risk could raise expected welfare. The reason for the latter result is that an increase in the risk of high or low damage costs can *reduce* the risk of being a signatory or non-signatory.

Full Learning

Again, the results are somewhat similar to the case with Risk Neutrality.

Result 6: Full Learning with Risk Aversion

If state $s = l,h$ *has been revealed at the outset, then in the emission and in the membership game nothing changes under risk aversion compared to risk neutrality (see Result 2). Taking account of the risk of being a coalition member or a fringe member in state s, the expected payoff per country in state s is* $V_{RA,FL}(s) = \rho(\gamma_s) = \omega(\gamma_s) - \alpha\delta(\gamma_s)$ *and the expected size of an IEA with Full Learning and Risk Aversion is* $n_{FL,RA} = pn_l + (1-p)n_h$*. Taking account of the risk of high or low damages, the ex-ante expected payoff per country is:*

$$V_{FL,RA} = p\rho(\gamma_l) + (1-p)\rho(\gamma_h) - \alpha\sqrt{p(1-p)[\rho(\gamma_l) - \rho(\gamma_h)]^2} \ .$$

Comparing the results for Risk Aversion with those with Risk Neutrality, we have in terms expected IEA size: $n_{FL,RA} = pn_l + (1-p)n_h = n_{FL,RN}$. Hence, risk aversion has no effect on the expected size of the IEA. In terms of expected welfare we have:

$$\begin{aligned} V_{FL,RA} &= [p\omega(\gamma_l) + (1-p)\omega(\gamma_h)] - \alpha[p\delta(\gamma_l) + (1-p)\delta(\gamma_h)] \\ &\quad -\alpha\sqrt{p(1-p)[\rho(\gamma_l) - \rho(\gamma_h)]^2} \leq [p\omega(\gamma_l) + (1-p)\omega(\gamma_h)] = V_{FL,RN} \qquad (11) \end{aligned}$$

Thus, expected welfare with Risk Aversion is lower than expected welfare with Risk Neutrality. This reflects the two sources of risk – the risks of being selected as signatories rather than non-signatories in the low and high damage cost states and the risk of the low or high damage cost states occurring.

More general comparative statics are more complicated than for the case of No Learning. What can be said is:

$$\frac{\partial n_{FL,RA}}{\partial x} = 0 \quad \text{where} \quad x = \alpha, \sigma(\gamma_s) \ . \qquad (12)$$

That is, the expected size of the IEA is unaffected by the degree of risk aversion and risk.

$$\begin{aligned} \frac{\partial V_{FL,RA}}{\partial \alpha} &= -[p\delta(\gamma_l) + (1-p)\delta\gamma_h] - \sqrt{p(1-p)[\rho(\gamma_l) - \rho(\gamma_h)]^2} \qquad (13) \\ &\quad + \alpha[\delta(\gamma_l) - \delta(\gamma_h)]\sqrt{p(1-p)} \end{aligned}$$

The first two terms in (13) are negative, while the third is ambiguous in sign. Thus, as in the case of No Learning, also under Full Learning, it is possible that an increase in risk aversion could raise expected welfare. We have not yet been able to analyse the effect on expected welfare of an increase in $\sigma(\gamma_s)$.

Partial Learning

Again, the results have a broad similarity to those of risk neutral case.

Result 7: Partial Learning and Risk Aversion

There are two possible stable IEAs.

(i) There is a stable IEA of size $n_{PL,RA} = n_h$ *where coalition members pollute in the low damage cost state and abate in the high damage cost state and fringe countries pollute in both states. The expected payoffs to coalition and fringe countries are given by:*

$$V^c_{RA,PL}(n_h) = -\hat{\gamma}N + p + (1-p)\gamma_h n_h - \alpha[1-\gamma_h n_h]\sqrt{p(1-p)} \text{ and}$$

$$V^f_{RA,PL}(n_h) = 1 - \hat{\gamma}N + (1-p)\gamma_h n_h + \alpha\gamma_h n_h\sqrt{p(1-p)}$$

(ii) There is a second stable IEA of size $n_{PL,RA} = n_l$ *iff* $p > \tilde{p} + \theta$ *with* $\theta \equiv \alpha\sqrt{p(1-p)}$ *where coalition members abate in both states and fringe countries pollute in both states. The expected payoffs to coalition and non-coalition countries are given by:* $V^c_{PL,RA} = -\hat{\gamma}(N - n_l)$ *and* $V^f_{PL,RA} = 1 - \hat{\gamma}(N - n)$.

(iii) If the selected IEA has $n_{PL,RA}$ *members, then the expected payoff per country is given by:*

$$V_{PL,RA} = \frac{n_{PL,RA}}{N} V^c_{PL,RA}(n_{PL,RA}) + \frac{(N - n_{PL,RA})}{N} V^f_{PL,RA}(n_{PL,RA})$$
$$- \frac{\alpha}{N}\sqrt{(N - n_{PL,RA})n_{PL,RA}[V^c_{PL,RA}(n_{PL,RA}) - V^f_{PL,RA}(n_{PL,RA})]^2}$$

The proof is given Appendix A. Note that there always exists an equilibrium $n_{PL,RA} = n_h$, but there may be a second equilibrium $n_{PL,RA} = n_l$ provided $p > \tilde{p} + \theta$ holds which however, as pointed out above already for the case of Risk Neutrality (which requires $p > \tilde{p}$ for this second equilibrium to exist), may be regarded as a not very interesting parameter constellation as the probability of the high damage cost state is very low. Like under Full Learning, Under Partial Learning the size of the coalition is not affected by risk and risk aversion. However, as $\theta \equiv \alpha\sqrt{p(1-p)} = \alpha\sigma(\gamma_s)/(\gamma_h - \gamma_l)$, the probability that the second equilibrium n_l occurs becomes lower with increasing risk and risk aversion.

The expected payoff per country in the equilibrium with $n_{PL,RA} = n_l$ are lower with Risk Aversion than Risk Neutrality as has been observed for No and Full Learning, and this is likely to be true for the equilibrium with $n_{PL,RA} = n_h$ for most plausible parameter values, though we have not been able to prove this for all parameter values (see Appendix). However, the effect of risk aversion and risk on the expected payoff per country is again not straightforward.

This concludes almost what we can say analytically about stable IEAs with Risk Aversion, except for one particular case in which a comparison among the three models of learning is straightforward.

Corollary 1: Comparison of No, Full and Partial Learning and Risk Aversion

Let $p = 0.5$ *and* $\alpha = 1$*, then the stable IEA for Partial Learning is* n_h *and for No learning* $\hat{n}$ *and* $n_h = \hat{n}$*. Moreover, expected payoff per country is the same for all three models of learning.*

Proof: Using (6), we have $\hat{\gamma} = 0.5(\gamma_l + \gamma_h) + 0.5(\gamma_h - \gamma_l) = \gamma_h$ and $n_h \equiv I(\gamma_h)$ and $\hat{n} \equiv I(\hat{\gamma})$. Moreover, $\theta = 0.5$ and hence $p < \tilde{p} + \theta$ and the second equilibrium $n_{PL,RA} = n_l$ does not exist. From Result 5, we have $V_{NL,RA} = \rho(\gamma_h)$, from Result 6 $V_{FL,RA} = 0.5[\rho(\gamma_l) + \rho(\gamma_h)] - 0.5[\rho(\gamma_l) - \rho(\gamma_h)] = \rho(\gamma_h)$ and from Result 7 $V^c_{PL,RA}(n_h) = -\gamma_h N + 0.5 + 0.5\gamma_h n_h - 0.5(1 - \gamma_h n_h) = -\gamma_h(N - n_h)$,
$V^f_{PL,NL}(n_h) = 1 - \gamma_h N + 0.5\gamma_h n_h + 0.5\gamma_h n_h = 1 - \gamma_h(N - n_h)$ and hence

$$V_{PL,RA}(n_h) = (N - n_h)(\frac{1}{N} - \gamma_h) - \frac{\sqrt{n_h(N - n_h)}}{N} = \rho(n_h).$$

In the next section, we will use some numerical analysis to first establish some of the properties of the stable IEAs with Risk Aversion and Partial Learning; we shall then use numerical analysis to compare the outcomes of the three models of learning with Risk Aversion.

3. Numerical Analysis

3.1 The Choice of Parameter Values

For the results that follow, we have fixed $N = 100$ and chosen 15 values of $\alpha =$ 0, 0.05, 0.10, 0.15, 0.20, 0.40, 0.60, … , 1.6, 1.8, 2.0 and 2.5. In Tables 1 and 3 we have conducted 500,000 Monte-Carlo simulations. For γ_s, $s = l, h$, we have chosen a uniform distribution of $1/\gamma_s$ between 1 and N, to ensure we get a corresponding uniform distribution of values for the corresponding sizes of IEA, $n_s = I(\gamma_s)$. We have chosen values of p from a uniform distribution lying between 0.02 and 0.98: we rule out very small probabilities because we want risk to be 'significant'. Following Assumption 2, we have to rule out some parameter combinations. In particular, we rule out parameter combination which result in any of the following: $\hat{\gamma} < 1$, $(n_l - \bar{n}) < 2$ and $(\bar{n} - n_h) < 2$. As noted in the discussion of Assumption 2, the first inequality ensures that in the No Learning case and Risk Aversion polluting is a dominant strategy in the Nash equilibrium and for non-signatories; the latter two inequalities ensure that uncertainty is sufficient to lead to distinct IEA sizes for n_l, $\bar{n}$ and n_h. How significant is the first restriction, in particular when compared to the case of Risk Neutrality?

Table 1 about here

Recall that Risk Neutrality required $\bar{\gamma} < 1$ (Assumption 1) and now with Risk Aversion $\hat{\gamma} < 1$ (Assumption 2) with $\hat{\gamma} \equiv \bar{\gamma} + \alpha\sigma(\gamma_s) \geq \bar{\gamma}$. The first column in Table 1 shows the proportion of the 500,000 combinations of parameter values that satisfy Assumption 2. We see that for $\alpha \leq 2.0$ approximately 63% of the 500,000 combinations of parameter values satisfy Assumption 2 and this percentage stays almost the same with increasing α. However, even for $\alpha > 2.0$, the proportion of accepted combinations of parameter values does fall only slightly. Thus, we conclude that the requirement $\hat{\gamma} \leq 1$ is not a significant restriction.[8]

[8] We have conducted simulations for a wider range of parameter values than those reported here – and even when $\alpha = 5.0$ the proportion of parameter values sampled which satisfy Assumption 2 drops only to 61.6%.

3.2 Stable IEAs for Partial Learning

The first question we need to resolve from the numerical analysis concerns what are the stable IEAs for the Partial Learning case. We showed in Result 7 that there is a critical value of p, such that if $p < \tilde{p} + \theta$, there is a unique stable IEA, namely n_h; while if $p \geq \tilde{p} + \theta$, n_l is also a stable IEA, and if we apply the selection criterion of maximum welfare (which is equivalent to Pareto dominance), then n_l will be selected rather than n_h. How frequently is the condition $p \geq \tilde{p} + \theta$ satisfied? The results are shown in the second column in Table 1.

We see that with Risk Neutrality (i.e. $\alpha = 0$) n_l is selected in only 0.57% of cases, and this falls quite sharply as α increases as the discussion of Result 7 suggested, falling to 0.01% by the time the coefficient of risk aversion reaches 2.0. So for the vast majority of parameter values the only stable IEA for Partial Learning is n_h.

3.3 How Risk Aversion Affects IEA Size and Welfare

Kolstad and Ulph (2008) showed that with Risk Neutrality there are two possible rankings of IEA size and welfare across No Learning (NL), Full Learning (FL) and Partial Learning (PL) as summarised in Result 4 above[9]: (i) $p \geq \tilde{p}$ then with PL there are two stable IEAs of which n_l is selected because of Pareto-dominance, and so $n_{NL,RN} \leq n_{FL,RN} \leq n_{PL,RN} = n_l$ while $V_{FL,RN} \leq V_{NL,RN} \leq V_{PL,RN}(n_l)$; (ii) in the much more likely outcome where $p < \tilde{p}$, with n_h the unique stable IEA with PL and $n_h = n_{PL,RN} \leq n_{NL,RN} \leq n_{FL,RN}$ while $V_{PL,RN}(n_h) \leq V_{FL,RN} \leq V_{NL,RN}$. It is worth noting in the latter case that there are two reasons why welfare is lower with PL than with NL: (i) there is a smaller number of signatories with PL than NL; (ii) with PL, in the low damage cost state of the world signatories pollute rather than abate.

How do these results change with risk aversion? We begin by looking at what happens for some specific parameter values and then conduct Monte Carlo simulations on a much wider range of parameter values.

[9] Recall that these results depend on ignoring the integer nature of IEA size, and so are strictly speaking approximations. See footnotes 5 and 6.

3.3.1 Specific Parameter Values

Tables 2(a) – (c) consider what happens as risk aversion increases for three different combinations of parameter values.

Tables 2(a) – (c) about here

Table 2(a): This case has quite large values for n_h and n_l but $0.5 = p < \tilde{p} < \tilde{p} + \theta$. Thus for PL, n_h is always the unique stable IEA and Risk Aversion has no effect on the size of IEA for FL and PL. For NL, with Risk Neutrality, IEA size lies between FL and PL, consistent with Kolstad and Ulph (2008), but as risk aversion increases, the size of IEA with NL falls as suggested by the discussion in section 2.2.3 (see equation (9), reaching n_h, the same as with PL, when $\alpha = 1$ (see Corollary 1), and then falls below n_h as risk aversion increases above 1. Now we can get parameter values with $n_{NL,RA} < n_h = n_{PL,RA} < n_{FL,RA}$, which was not possible with Risk Neutrality.

For welfare, there are several effects of increasing risk aversion on welfare. First, there is simply the direct effect that as risk aversion increases, welfare falls even if there is no change in number of signatories or emissions. To isolate this effect, we have also computed welfare for the non-cooperative equilibrium apart from the coalitional equilibrium (i.e. IEA) for NL, FL and PL. We compute both the per country level of welfare and the proportionate gain in welfare (expressed in percentages), denoted *g* from an IEA relative to the non-cooperative equilibrium.

As risk aversion increases, the gain in welfare also falls for all three models of learning. When $\alpha = 2.5$, we have effectively exhausted the gains from IEAs for all forms of learning. Why is this?

Consider first FL. As risk aversion increases, there is neither a change to the number of signatories in each state of the world nor to their emissions. So why does welfare fall faster than in the non-cooperative equilibrium? With FL, IEAs create two risks in addition to the risk of high or low damage costs: (i) when the state of the world changes from low damage to high damage the number of signatories *falls*; this is the usual 'perverse' behaviour story but it increases the risk attached to having a high damage cost; (ii) there is now the risk associated with being a signatory or a non-signatory –

which is not an issue in the non-cooperative world. As risk aversion increases, these additional risks that arise with an IEA reduce the welfare gains from forming an IEA relative to the non-cooperative outcome.

Now consider NL. Because the number of signatories is the same in both states of the world, the first factor in FL does not apply. However the second additional risk – of being a signatory or non-signatory – still applies. But now there is also now the direct effect that the number of signatories falls as risk aversion increases, reducing the welfare gains relative to the non-cooperative equilibrium.

Finally, consider PL. As with NL, the number of signatories does not change with the state of the world. Thus, the first source of additional risk with FL is not present with PL. Unlike NL, for the parameter values considered in Table 2(a), the number of signatories does not change as risk aversion increases. However, there is another factor with PL which affects the welfare cost of risk: signatories abate in the high damage cost state and pollute in the low damage cost state, which reduces the effect of risk.

It is clear that there are different factors affecting the welfare of countries as risk aversion increases under the three different models of learning. Therefore, it is not surprising that welfare ranking across the three different models change as risk aversion increases. With Risk Neutrality, the ranking is as predicted by Kolstad and Ulph (2008) – welfare is higher with NL than with FL which is higher than with PL. This ranking remains the same until $\alpha = 1$, when, as proved in Corollary 1, welfare is the same in all models of learning. For $\alpha > 1$, welfare is now highest with FL, and for α between 1.2 and 1.6, welfare is higher with PL than with NL.

In summary, while it is straightforward to understand what happens to the size of IEA under the three regimes of NL, FL and PL as risk aversion increases, there are interesting complex effects on welfare because of the different elements of risk, and so the rankings of welfare between the three learning regimes change in complex ways.

<u>Table 2 (b):</u> In Table 2(b) we keep $p = 0.5$, but choose higher values of damage costs and hence lower values of n_l and n_h. The gains from learning are smaller than in Table 2(a) because of the lower sizes of stable IEAs. One interesting difference compared to

Table 2(a) is that while the relative welfare gains from an IEA relative to the non-cooperative outcome (i.e. *g*) fall as risk aversion increases for NL and FL, the gains with PL increase slightly with α for $\alpha < 1$. The welfare gains from NL are higher than FL for $\alpha < 0.2$, are higher with FL than NL for $\alpha > 1.6$, and switch around for intermediate values of α. The interesting difference now is that, while the welfare gains from PL are lower than with NL or FL for $\alpha < 1$, as with Table 2(a), they are higher than with NL or FL for $\alpha > 1$.

Table 2 (c): In this table, we keep the same values for damage costs as in Table 2(b) but choose a much higher value for *p* such that $p > \tilde{p}$. The key difference to Table 2(b) is that with low risk aversion, with PL n_l is the selected stable IEA (because $p > \tilde{p} + \theta$), and so for these values of risk aversion the number of signatories and welfare are higher with PL than with either NL or FL. But when $\alpha > 0.1$, the stable IEA switches to n_h (because $p < \tilde{p} + \theta$) and now welfare with PL lies below FL and NL for all higher values of α, even though the relative welfare gain with PL increases for $0.1 < \alpha < 2.0$. The reason is that for $\alpha > 0.1$ the IEA size with PL is always lower than with NL or FL. Comparing NL and FL, we see that for $\alpha < 0.6$, FL has higher welfare than NL[10], but for higher values of α, the relative welfare gains are very similar and so the ranking between NL and FL switches around.

3.3.2 Results from Monte Carlo Simulations

The analysis in section 3.3.1 was for 3 specific sets of parameter values other than α We now present the results of 500,000 Monte Carlo simulations conducted on the basis set out at the beginning of section 3. The results are shown in Table 3(a) for size of IEA and Table 3(b) for welfare, respectively. For each of the valid combinations of parameter values the programme calculated the ranking of the relevant variable across the three models of learning. There are 13 possible rankings, and the programme then calculated the proportion of all valid parameter combinations which satisfied each of these possible rankings. Only seven of the 13 possible rankings had non-zero values for at least one of the two variables for some values of α and these proportions are

[10] Note that this applies even for $\alpha = 0$, contrary to the result of Kolstad and Ulph (2008), but this is due to the approximation of treating IEA size as a real number not an integer. See footnote 9.

reported in the first seven columns of Tables 3(a) and 3(b). To assess the results more readily, the final three columns present the proportion of valid parameter values for which FL, PL and NL are the highest ranked.

Table 3(a) and (b) about here

Table 3(a) Size of IEA

As already noted in the discussion of Table 1, the analysis of Risk Neutrality in Kolstad and Ulph (2008) showed that the ranking of IEA size would be as in Case 1 except for the small proportion of cases (just over 0.5%) for which n_l would be selected as the stable IEA for PL, in which case the ranking in Case 5 would be selected. Table 1 showed that the proportion of cases for which n_l is selected as the stable IEA for PL falls rapidly as risk aversion increases. These results are largely confirmed by Table 3(a) except that there is a small proportion of cases (~3%) for which with risk neutrality the ranking in Case 7 arises. As noted in footnotes 9 and 10, this reflects the fact that the result in Kolstad and Ulph (2008) relies on an approximation. As risk aversion increases this case rapidly disappears. So for $\alpha > 0.1$, the largest expected size of IEA always arises with FL. However, as columns displaying the proportion of Cases 1 to 3 show, the ranking of expected IEA size for NL and PL changes quite sharply as α rises above 0.2. There is now a rapidly increasing number of cases for which expected IEA size with No Learning is below that for Partial Learning (i.e. Case 2), reflecting the fact that as risk aversion increases the size of IEA with NL falls while that for PL is constant.

Table 3(b) Welfare

Again starting from Risk Neutrality, the analysis of Kolstad and Ulph (2008) showed that to a first approximation we would expect the welfare ranking given by Case 7 except for the small proportion of cases for which with PL n_l is the selected stable IEA. The simulation results show that these results are again a first approximation, and there are about 5% of cases for which (as in Table 2(c)) welfare is higher with FL than NL (i.e. Case 1).

As risk aversion increases the proportion of cases for which NL generates the highest welfare falls as risk aversion increases, falling quite slowly initially and remaining above 0.5 for risk aversion below 1.0; but once risk aversion rises above 1 the proportion of cases for which NL generates the highest welfare falls quite sharply (see Figure 1).

Figure 1 about here

By contrast the proportion of cases for which FL generates the highest welfare rises as risk aversion increases, rising quite slowly for risk aversion less than 1 but more rapidly as risk aversion increases above 1 until for $\alpha > 1.6$ the proportion of parameter combinations for which FL generates the highest level of welfare is higher than for PL or NL (see Figure 1). Finally, for PL, as risk aversion increases above 0.2 there is an initially growing proportion of cases for which welfare is highest, and for α in the range 1.2 – 1.6 the proportion of parameter combinations for which PL yields the highest welfare is greater than for FL or NL (see Figure 1). All together, it is evident that with increasing risk aversion, learning, either in the form of Partial or Full learning, yields higher welfare than No Learning.

4. Summary and Conclusions

Kolstad and Ulph (2008) showed that with risk neutrality the possibility of learning better information about environmental damage costs in the future had rather pessimistic implications for the formation of IEAs. Except for a very small set of parameter values for which partial learning would select a high IEA membership, learning resulted in lower expected membership for partial learning and lower expected welfare for both full and partial learning. Hence, in a strategic context, learning is bad.

This result is qualified when one takes account of risk aversion. As risk aversion increases, the proportion of cases for which expected membership is higher with no learning than partial learning falls steadily, and when risk aversion is sufficiently high, there are more parameter combinations for which membership with partial learning is higher than with no learning. In terms of expected welfare the proportion of parameter values for which no learning yields higher expected welfare than partial learning or full learning falls as risk aversion increases and for sufficiently high risk aversion this relation is reversed.

However, while increasing risk aversion makes learning – either in the form of full or partial learning – more attractive from global welfare perspective, this is in the context in which the gains from forming an IEA relative to the non-cooperative equilibrium fall constantly and become very small (2% -3%).

Thus, we can add two versions of the paradox of the commons known from Barrett (1994). Whenever the relative gains from stable cooperation would be large, learning has a negative impact on the success of IEAs. The more risk averse governments are, the lower are the relative gains from stable cooperation.

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Appendix A: Proof of Result 7

<u>Emissions Game</u>

The strategies are the same as for Result 3, but the expected payoffs are different because of risk aversion. Fringe countries always pollute no matter the state of the world or the number of coalition members. The strategy for coalition members depends on the number of members and the state of the world as follows:

(i) $n \geq \frac{1}{\gamma_l}$: coalition members abate in both states of the world. Expected payoffs are:

$V^c(n) = -\hat{\gamma}(N-n)$ and $V^f(n) = 1 - \hat{\gamma}(N-n)$.

(ii) $n < \frac{1}{\gamma_h}$: coalition members pollute in both states of the world. Expected payoffs are:

$V^c(n) = V^f(n) = 1 - \hat{\gamma}N$.

(iii) $\frac{1}{\gamma_h} \leq n < \frac{1}{\gamma_l}$: coalition members pollute in the low damage cost state and abate in high damage cost state. Payoffs in the two states are:

Low damage cost: $V_l^c = V_l^f = 1 - \gamma_l N$;

High damage cost: $V_h^c = -\gamma_h(N-n)$ and $V_h^f = 1-\gamma_h(N-n)$.

Expected payoffs, taking account of risk aversion, are:

$$
\begin{aligned}
V^c(n) &= [pV_l^c + (1-p)V_h^c] - \alpha\sqrt{p(1-p)(V_l^c - V_h^c)^2} \\
&= -\overline{\gamma}N + p + (1-p)\gamma_h n - \alpha\sqrt{p(1-p)}[1+(\gamma_h-\gamma_l)N - \gamma_h n] \\
&= -\hat{\gamma}N + p + (1-p)\gamma_h n - \alpha\sqrt{p(1-p)}[1-\gamma_h n]
\end{aligned}
$$

$$
\begin{aligned}
V^f(n) &= [pV_l^f + (1-p)V_h^f] - \alpha\sqrt{p(1-p)(V_l^f - V_h^f)^2} \\
&= 1-\overline{\gamma}N + (1-p)\gamma_h n - \alpha\sqrt{p(1-p)}[(\gamma_h-\gamma_l)N - \gamma_h n] \\
&= 1-\hat{\gamma}N + (1-p)\gamma_h n + \alpha\sqrt{p(1-p)}[\gamma_h n]
\end{aligned}
$$

Membership Game

Using the expected payoffs calculated above, we now need to determine stable IEAs. Recall that with Risk Neutrality we showed in Result 3 that n_h is always a stable IEA while if $p \geq \tilde{p}$, then n_l is also stable. We now analyze stability for the Partial Learning with Risk Aversion, using the payoff functions derived above.

a) We shall ignore $n \leq n_h - 1$ which generates trivial stable IEAs where all countries pollute.

b) No $n > n_l$ can be stable because $V^c(n) - V^f(n-1) = -\hat{\gamma}(N-n) - 1 + \hat{\gamma}(N-n+1) = \hat{\gamma} - 1 < 0$ (where we use i) from above).

c) No n such that $n_h < n < n_l$ can be stable. We have (where we use iii) from above):

$V^c(n) = -\overline{\gamma}N + p + (1-p)\gamma_h n - \alpha\sqrt{p(1-p)}[1+(\gamma_h-\gamma_l)N - \gamma_h n]$ and

$V^f(n-1) = 1-\overline{\gamma}N + (1-p)\gamma_h(n-1) - \alpha\sqrt{p(1-p)}[(\gamma_h-\gamma_l)N - \gamma_h(n-1)]$ and

therefore $V^c(n) - V^f(n-1) = (\gamma_h - 1)(1-p+\alpha\sqrt{p(1-p)}) < 0$.

Thus, the analysis so far just leaves 2 candidates for (non-trivial) stable IEAs: n_l and n_h.

d) When is n_l stable? It will save notation to define $\theta \equiv \alpha\sqrt{p(1-p)}$. From b) we know that $V^c(n_l+1)<V^f(n_l)$, so n_l is externally stable. It will be internally stable iff $V^c(n_l)\geq V^f(n_l-1)$ (where we use i) from above for $n=n_\ell$ for the LHS and iii) for the RHS of this inequality).

$$\begin{aligned}
&\Leftrightarrow -\hat{\gamma}(N-n_l)\geq 1-\hat{\gamma}N+(1-p)\gamma_h(n_l-1)+\gamma_h(n_l-1)\theta\\
&\Leftrightarrow \hat{\gamma}n_l-1-(1-p)\gamma_h(n_l-1)-\gamma_h(n_l-1)\theta\geq 0\\
&\Leftrightarrow p\gamma_l n_l+(1-p)\gamma_h n_l+(\gamma_h-\gamma_l)\theta n_l-1-(1-p)\gamma_h(n_l-1)-\gamma_h(n_l-1)\theta\geq 0\\
&\Leftrightarrow p(1+\varepsilon\gamma_l-\gamma_h)-(1-\gamma_h)+(\gamma_h-\gamma_l)\theta n_l-\gamma_h(n_l-1)\theta\geq 0\\
&\Leftrightarrow p(1+\varepsilon\gamma_l-\gamma_h)-\tilde{p}(1-\gamma_h+\varepsilon\gamma_l)+\theta[\gamma_h-n_l\gamma_l]\geq 0 \qquad \text{(I)}\\
&\Leftrightarrow (p-\tilde{p})(1+\varepsilon\gamma_l-\gamma_h)\geq\theta[n_l\gamma_l-\gamma_h]\\
&\Leftrightarrow (p-\tilde{p})(1+n_l\gamma_l-1-\gamma_h)\geq\theta[n_l\gamma_l-\gamma_h]\\
&\Leftrightarrow (p-\tilde{p})\geq\theta\\
&\Leftrightarrow p\geq\tilde{p}+\theta
\end{aligned}$$

Now with Risk Neutrality, i.e. $\theta=0$, and condition (I) is just the condition that n_l is stable, i.e. $p\geq\tilde{p}$, as in Result 3. For Risk Aversion, i.e. $\theta>0$, we can conclude as for Risk Neutrality that if $p\leq\tilde{p}$, the inequality is not satisfied for risk averse players $(\alpha>0)$ and n_l is not stable. A necessary so not sufficient condition for n_l to be stable is therefore $p>\tilde{p}$.

e) When is n_h stable? We know from c) that $V^c(n_h+1)<V^f(n_h)$ and so n_h is externally stable. It will be internally stable if $V^c(n_h)\geq V^f(n_h-1)$ (where use for the LHS of this inequality iii) and for the RHS ii) from above):

$$\begin{aligned}
&\Leftrightarrow -\hat{\gamma}N+p+(1-p)\gamma_h n_h-\theta[1-\gamma_h n_h]\geq 1-\hat{\gamma}N\\
&\Leftrightarrow (1-p)(\gamma_h n_h-1)\geq\theta[1-\gamma_h n_h]\Leftrightarrow 1-p\geq-\theta \qquad \text{(II)}\\
&\Leftrightarrow \theta\geq p-1
\end{aligned}$$

As $\theta\geq 0$, condition (II) always holds. Thus n_h is always an equilibrium.

To summarise the analysis, as in the case of risk neutrality, the only possible candidates for stable IEAs are n_l and n_h. n_h is always a stable equilibria and n_l if $p>\tilde{p}$ and condition (I) holds. Hence, risk has no effect on the equilibrium size of the coalition. As

$\theta \equiv \alpha\sqrt{p(1-p)} = \alpha\sigma(\gamma_s)/(\gamma_h - \gamma_l)$, the likelihood that n_l is an equilibrium decreases the higher risk and risk aversion.

To complete the analysis we need to compute per country welfare for Partial Learning. If the selected IEA has n_{PL} signatories, then the expected per country welfare from Partial Learning is:

$$V_{PL,RA} = \frac{n_{PL}}{N} V^c_{PL,RA}(n_{PL}) + \frac{(N - n_{PL})}{N} V^f_{PL,RA}(n_{PL}) - \frac{\alpha}{N}\sqrt{(N - n_{PL})n_{PL}[V^c_{PL}(n_{PL}) - V^f_{PL}(n_{PL})]^2}$$

In the case of n_l, we get:

$$\begin{aligned} V_{PL,RA}(n_l) &= -\frac{n_l}{N}\hat{\gamma}(N-n_l) + \left(1-\frac{n_l}{N}\right)\left(1-\hat{\gamma}(N-n_l)\right) - \alpha\sqrt{\left(1-\frac{n_l}{N}\right)\frac{n_l}{N}} \\ &= 1-\hat{\gamma}(N-n_l) - \frac{n_l}{N} - \alpha\sqrt{(1/N\gamma_l)[1-(1/N\gamma_l)]} \\ &= 1-\hat{\gamma}(N-n_l) - \frac{n_l}{N} - \alpha\delta(\gamma_l) \end{aligned}$$

Using $\hat{\gamma} \equiv \overline{\gamma} + \alpha\sigma(\gamma_s)$, $V_{PL,RA}(n_l) = 1-\overline{\gamma}(N-n_l) - \frac{n_l}{N} - \alpha\omega(\gamma_l) - \alpha\sigma(\gamma_s)(N-n_l)$ which is smaller than $V_{PL,RN}(n_l) = 1-\overline{\gamma}(N-n_l) - \frac{n_l}{N}$ as stated in Result 3. Moreover,

$$\frac{\partial V_{PL,RA}(n_l)}{\partial \alpha} = -(N-n_l)\sigma(\gamma_s) - \delta(\gamma_l) < 0$$

and

$$\frac{\partial V_{PL,RA}(n_l)}{\partial \sigma(\gamma_s)} = -(N-n_l)\alpha < 0 .$$

In the case of n_h, we get:

$$V_{PL,RN}(n_h) = (1-N\overline{\gamma}) + (1-p)n_h(\gamma_h - 1/N)$$

$$V_{PL,RA}(n_h) = \frac{n_h}{N} V^c_{PL,RA}(n_h) + \frac{N-n_h}{N} V^f_{PL,RA}(n_h) - \frac{\alpha}{N}\sqrt{(N-n_h)n_h\left[V^f_{PL,RA}(n_h) - V^c_{PL,RA}(n_h)\right]^2}$$

$$= \frac{n_h}{N} V^c_{PL,RA}(n_h) + \left(1-\frac{n_h}{N}\right) V^f_{PL,RA}(n_h) - \alpha\delta(\gamma_h)\left(V^f_{PL,RA}(n_h) - V^c_{PL,RA}(n_h)\right)$$

$$= V^f_{PL,RA}(n_h) + \left(V^f_{PL,RA}(n_h) - V^c_{PL,RA}(n_h)\right)\left(-\alpha\delta(\gamma_h) - \frac{n_h}{N}\right)$$

$$= V^f_{PL,RA}(n_h) - \left(\alpha\delta(\gamma_h) + \frac{n_h}{N}\right)\left((1-p) + \alpha\sqrt{p(1-p)}\right)$$

$$= 1 - \overline{\gamma}N + (1-p)[\gamma_h n_h - n_h / N] + \alpha\sqrt{p(1-p)}\gamma_h n_h - \alpha(1-p)\delta(\gamma_h) - \alpha\sqrt{p(1-p)}[\alpha\delta(\gamma_h) + \frac{n_h}{N}]$$

So we can conclude:

$$V_{PL,RA}(n_h) \leq V_{PL,RN}(n_h) \Leftrightarrow n_h(\gamma_h - \frac{1}{N}) < \delta(\gamma_h)\left[\alpha + \sqrt{\frac{1-p}{p}}\right] \quad \text{(III)}$$

However, even allowing for the fact that $p \leq \tilde{p} \leq 1$, it is clear that we can choose parameter values for $\alpha \cong 0$ and $p \cong \tilde{p} \cong 1$ such the LHS of the inequality (III) is strictly positive while the RHS is approximately zero. So there could be some extreme parameter values for which the inequalities in (III) do not hold, though for almost all parameter values we would expect the inequalities in (III) to hold.

Table 1: Percentage of Usable Simulation Runs and Percentage of Partial Learning Cases with 2 Stable IEAs

	Percentage of Usable Simulation Runs	Percentage of Usable Simulation Runs for which n_l is stable
0.00	63.2	0.57
0.05	63.0	0.46
0.10	63.1	0.37
0.15	63.1	0.30
0.20	63.0	0.26
0.30	63.2	0.20
0.40	63.1	0.15
0.50	63.1	0.13
0.60	63.0	0.10
0.70	63.1	0.09
0.80	63.2	0.06
0.90	63.1	0.06
1.00	63.2	0.06
1.20	63.0	0.05
1.40	63.0	0.04
1.60	63.2	0.03
1.80	63.2	0.02
2.00	63.1	0.01
2.50	62.8	0.01

Table 2: Comparison of No Learning, Full Learning and Partial Learning for Specific Parameter Values

2(a) Parameter Values: γ_h =0.0687; γ_l = 0.0164; p = 0.5; $\bar{\gamma}$ = 0.0426; n_l = 61; n_h =15; $\bar{n}$ =24

α	No Cooperation	IEA No Learning			IEA Full Learning			IEA Partial Learning		
	V	n^*	V	g	n^*	V	g	n^*	V	g
0.00	-3.255	24	-2.474	24.0	38	-2.620	19.5	15	-2.815	13.5
0.05	-3.386	23	-2.628	22.4	38	-2.759	18.5	15	-2.933	13.4
0.10	-3.517	23	-2.750	21.8	38	-2.898	17.6	15	-3.052	13.2
0.15	-3.647	22	-2.907	20.3	38	-3.037	16.7	15	-3.172	13.0
0.20	-3.778	21	-3.066	18.88	38	-3.175	15.9	15	-3.293	12.8
0.40	-4.301	19	-3.641	15.4	38	-3.726	13.4	15	-3.785	12.0
0.60	-4.824	18	-4.186	13.2	38	-4.271	11.5	15	-4.291	11.0
0.80	-5.347	16	-4.785	10.5	38	-4.812	10.0	15	-4.812	10.0
1.00	-5.870	15	-5.347	8.9	38	-5.347	8.9	15	-5.347	8.9
1.20	-6.393	14	-5.914	7.5	38	-5.876	8.1	15	-5.896	7.8
1.40	-6.916	13	-6.488	6.2	38	-6.401	7.4	15	-6.459	6.6
1.60	-7.439	12	-7.066	5.0	38	-6.920	7.0	15	-7.037	5.4
1.80	-7.962	12	-7.592	4.7	38	-7.434	6.6	15	-7.629	4.2
2.00	-8.485	11	-8.177	3.6	38	-7.943	6.4	15	-8.235	2.9
2.50	-9.793	10	-9.563	2.3	38	-9.192	6.1	15	-9.814	-0.2

V=expected per country payoff; n^* = equilibrium coalition size; g =relative gain (%) compared to No Cooperation.

Table 2: Comparison of No Learning, Full Learning and Partial Learning for Specific Parameter Values

2(b) Parameter Values: γ_h =0.2198; γ_l = 0.0909; p = 0.5; $\bar{\gamma}$ = 0.1554; n_l = 12; n_h =5; $\bar{n}$ = 7

α	No Cooperation	IEA No Learning			IEA Full Learning			IEA Partial Learning		
	V	n^*	V	g	n^*	V	g	n^*	V	g
0.00	-14.534	7	-13.517	7.0	8.5	-13.524	6.9	5	-14.011	3.6
0.05	-14.856	7	-13.829	6.9	8.5	-13.858	6.7	5	-14.312	3.7
0.10	-15.178	7	-14.141	6.8	8.5	-14.191	6.3	5	-14.615	3.7
0.15	-15.501	7	-14.454	6.8	8.5	-14.524	6.1	5	-14.917	3.8
0.20	-15.823	6	-14.921	5.7	8.5	-14.857	6.1	5	-15.221	3.8
0.40	-17.112	6	-16.180	5.4	8.5	-16.186	5.4	5	-16.440	3.9
0.60	-18.400	6	-17.439	5.2	8.5	-17.511	4.8	5	-17.667	4.0
0.80	-19.689	5	-18.879	4.1	8.5	-18.831	4.4	5	-18.904	4.0
1.00	-20.978	5	-20.147	4.0	8.5	-20.147	4.0	5	-20.147	4.0
1.20	-22.267	5	-20.415	3.8	8.5	-21.459	3.6	5	-21.403	3.9
1.40	-23.556	5	-22.683	3.7	8.5	-22.766	3.4	5	-22.665	3.8
1.60	-24.844	4	-24.164	2.7	8.5	-24.069	3.1	5	-23.937	3.7
1.80	-26.133	4	-25.441	2.7	8.5	-25.368	2.9	5	-25.217	3.5
2.00	-27.422	4	-26.717	2.6	8.5	-26.663	2.8	5	-26.505	3.4
2.50	-30.644	4	-29.908	2.4	8.5	-29.881	2.5	5	-29.765	2.99

V=expected per country payoff; n^* = equilibrium coalition size; g =relative gain (%) compared to No Cooperation.

Table 2: Comparison of No Learning, Full Learning and Partial Learning for Specific Parameter Values

2 (c) Parameter Values: γ_h =0.2198; γ_l = 0.0909; p = 0.925; $\bar{\gamma}$ = 0.1006; n_l = 12; n_h =5; $\bar{n}$ =10

α	No Cooperation	IEA No Learning			IEA Full Learning			IEA Partial Learning		
	V	n^*	V	g	n^*	V	g	n^*	V	g
0.00	-9.057	10	-8.151	10.0	11.48	-8.080	10.8	12	-7.970	12.0
0.05	-9.227	10	-8.319	9.8	11.48	-8.265	10.4	12	-8.136	11.8
0.10	-9.396	10	-8.487	9.7	11.48	-8.449	10.1	12	-8.301	11.7
0.15	-9.566	10	-8.654	9.5	11.48	-8.633	9.8	5	-9.450	1.2
0.20	-9.736	10	-8.822	9.4	11.48	-8.817	9.4	5	-9.607	1.3
0.40	-10.415	9	-9.592	7.9	11.48	-9.552	8.3	5	-10.241	1.7
0.60	-11.094	9	-10.267	7.5	11.48	-10.285	7.3	5	-10.880	1.9
0.80	-11.773	8	-11.048	6.2	11.48	-11.015	6.4	5	-11.523	2.1
1.00	-12.452	8	-11.727	5.8	11.48	-11.743	5.7	5	-12.171	2.3
1.20	-13.131	8	-12.406	5.5	11.48	-12.469	5.0	5	-12.823	2.3
1.40	-13.810	7	-13.200	4.4	11.48	-13.193	4.5	5	-13.480	2.4
1.60	-14.489	7	-13.883	4.2	11.48	-13.914	4.0	5	-14.141	2.4
1.80	-15.168	7	-14.566	4.0	11.48	-14.633	3.5	5	-14.807	2.4
2.00	-15.847	6	-15.371	3.0	11.48	-15.350	3.1	5	-15.478	2.4
2.50	-17.545	6	-17.086	2.6	11.48	-17.133	2.3	5	-17.175	2.1

V=expected per country payoff; n^* = equilibrium coalition size; g =relative gain (%) compared to No Cooperation.

Table 3a: Percentages of Cases with Ranking of IEA Size over No Learning, Full Learning and Partial Learning

α	Case Number							Highest Ranked		
	1	2	3	4	5	6	7	FL	PL	NL
0.00	96.4	0	0	0	0.6	0	3.0	96	1	3
0.05	98.0	0	0	0	0.4	0	1.6	98	0	2
0.10	98.9	0	0	0	0.4	0	0.8	99	0	1
0.15	99.4	0	0	0	0.3	0	0.3	100	0	0
0.20	99.4	0	0.2	0	0.3	0	0.2	100	0	0
0.40	92.4	0.4	7.1	0	0.2	0	0	100	0	0
0.60	78.1	6.7	15.1	0	0.1	0	0	100	0	0
0.80	63.0	18.6	18.4	0	0.1	0	0	100	0	0
1.00	50.0	31.4	18.6	0	0.1	0	0	100	0	0
1.20	39.2	43.9	16.9	0	0	0	0	100	0	0
1.40	31.0	54.4	14.7	0	0	0	0	100	0	0
1.60	24.6	62.9	12.5	0	0	0	0	100	0	0
1.80	19.6	69.8	10.6	0	0	0	0	100	0	0
2.00	15.8	75.3	8.8	0	0	0	0	100	0	0
2.50	9.5	84.5	6.0	0	0	0	0	100	0	0

Case 1: FL > NL > PL; Case 2: FL > PL > NL; Case 3: FL > NL = PL;

Case 4: PL > NL > FL; Case 5: PL > FL > NL; Case 6: NL > PL > FL; Case 7: NL > FL > PL.

Highest Ranked: FL: Cases 1 + 2 +3; PL: Cases 4+5; NL: Cases 6+7

Table 3b: Percentages of Cases with Ranking of Welfare over No Learning, Full Learning and Partial Learning

α	Case Number							Higest Ranked		
	1	2	3	4	5	6	7	FL	PL	NL
0.00	5.4	0	0	0.3	0.2	0	94.1	5	1	94
0.05	6.1	0	0	0.3	0.2	0	93.5	6	0	94
0.10	6.8	0	0	0.3	0.1	0	92.8	7	0	93
0.15	7.4	0	0	0.2	0.1	0	92.3	7	0	92
0.20	7.9	0.1	0	0.2	0.1	0	91.5	8	0	92
0.40	11.0	1.8	0	0.8	1.4	7.8	77.2	13	2	85
0.60	12.2	4.7	0	4.6	7.2	14.6	56.8	17	12	71
0.80	12.0	8.8	0	10.5	13.9	16.5	38.4	21	24	55
1.00	12.8	12.3	0	16.2	18.	15.3	25.4	25	39	41
1.20	15.4	14.9	0	19.8	19.6	12.9	17.4	30	40	30
1.40	19.4	16.0	0	21.6	19.3	10.5	13.2	35	41	24
1.60	23.9	16.3	0	22.3	17.8	8.4	11.4	40	40	20
1.80	28.8	15.7	0	21.9	15.8	6.8	11.0	45	38	18
2.00	33.5	14.9	0	21.3	13.8	5.4	11.2	48	35	17
2.50	43.2	11.6	0	19.7	8.9	3.4	13.3	55	28	17

Case 1: FL > NL > PL; Case 2: FL > PL > NL; Case 3: FL > NL = PL;

Case 4: PL > NL > FL; Case 5: PL > FL > NL; Case 6: NL > PL > FL; Case 7: NL > FL > PL.

Highest Ranked: FL: Cases 1 + 2 +3; PL: Cases 4+5; NL: Cases 6+7

Figure 1: Highest Ranking of Welfare over No Learning, Full Learning and Partial Learning