A Parameterized Microwace Emission Model for Dry Snow Cover

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***Abstract* – In order to develop retrieval algorithm for snow depth or snow water equivalence (SWE), we carried out the sensitivity test between the emission models with the different scattering-order: the zeroth-order, the first-order and the multi-scattering models. The multi-scattering microwave emission model used in this study includes the Dense Media Radiative Transfer Model (DMRT) and AIEM to simulation of dry snow emission with Matrix Doubling approach. The comparison results indicated that the multi-scattering effects have to be taken into account in the snow emission model, especially for large grain size. Due to the complexity of the multi-scattering model, we developed a parameterized inversion model using our multi-scattering emission model with a wide range of snow and underground properties for algorithm development purpose.**

Keywords- snow, passive microwave remote sensing, parameterization

I. INTRODUCTION

Microwave remote sensing techniques have capabilities to provide the distributed snow properties. Measurement of the amount of water stored in the snowpack and forecasting the rate of melt are thus essential for management of water supply. Because microwave radiation penetrates through snow, microwave remote sensing retrieval of snow parameters, such as snow extent, snow water equivalent, and wet/dry state, has been investigated by many researchers using various microwave sensors with various degree of success. At coarse resolution, for global and continental scale, snow has been a focus of passive microwave remote sensing. Passive microwave remote sensing can provide useful information at large scale on snow cover characteristics for hydrological, climatic, and meteorological applications.

Microwave brightness temperature measurements between 3 GHz and 90 GHz have found sensitive to snow type and water equivalent [1]. At the lower frequencies of the microwave band, emission from a dry snow cover is mainly affected by underlying soil dielectric and roughness properties. At the higher frequencies, however, emission is sensitive to snow water equivalence and snow particle size since the volume scattering by snow particles becomes important. Because dry snow emits considerably less microwave radiation than soil, the brightness temperature of snow is inversely related to the snow water equivalence, and the measurements can be carried out through cloud cover. When snow starts to melt, emission will significantly increase due to the high dielectric contrast between ice and liquid water in microwave spectrum and the observed signals are only emitted from the near snow surface [2].

II. MODEL DESCRIPTION

The radiative transfer model can be solved exactly by numerical methods, such as Matrix Doubling method [3], [4] or the eigen-analysis technique [4]. For our snow emission mode, the Matrix Doubling approach is used to solve the vector radiative transfer equations to include the multi-scattering effects. This model uses 1) the dense media model (DMRT) with the Mie scattering assumption[5] to describe snow pack extinction and emission properties and 2) the Advanced Integral Equation Model (AIEM) [6] to calculate the subsurface emission signal, and to calculate the boundary conditions at the snow-air and snow-soil interfaces for the vector radiative transfer model. The detail description of our snow emission model is in [8].

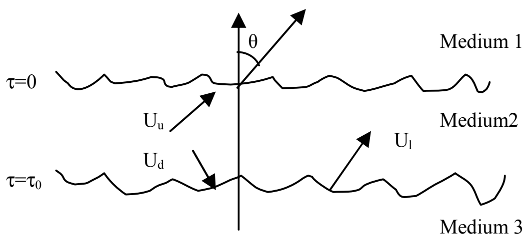

Fig.1 Geometry of the single layer emission problem.

A. The Matrix Doubling method

The Matrix Doubling approach which considers multi-scattering effects is a numerical solution to radiative transfer equations. There were three major emission sources for an inhomogeneous layer: upwelling and downwelling of emitted intensities within the snow layer; upward emission from the lower half space. These intensities were expected to go through some or all of the following processes, volume scattering, surface scattering, attenuation, surface-volume interactions and transmission across irregular boundary or boundaries, before they arrive at the receiver.

The total emission u_t into the mdeium due to u_u, u_d and u_l may be written

$$u_t = L_u u_u + L_d u_d + L_l u_l \tag{1}$$

where L's are the multiple scattering operators.

u_u,: the total upward emission from the layer.

u_d: the total downward emission from the layer.

u_l: the emission from the lower homogeneous half space.

The matrix-doubling method provides an alternative to the radiative transfer method for computing the combined scattering effects of surface and volume scattering. It is also based on energy balance and has been shown to be an equivalent formulation to the radiative transfer approach [7]. In actual computation this is a more efficient method for layers having large optical thickness.

B. The Advanced Integral Equation Method (AIEM)

At the interface between two homogeneous media, the scattering characteristics are determined by the interface roughness and the discontinuity between the media. Several surface scattering models have been developed in the last three decades. They are the small perturbation model (SPM), Kirchhoff model (KM), phase perturbation model (PPM), full wave model (FWM), and the integral equation model (IEM). The IEM model was verified by laboratory measurements of bistatic scattering from surfaces with small, intermediate and large scale roughness. And the advanced IEM (AIEM) kept the absolute phase term in Greens function, meaning which has more accuracy than the old IEM [6]. So we applied AIEM to deal with boundary effects in this study.

C. The zeroth-order and first-order solutions

In order to do different-scattering order comparison, here we give the expression of zeroth-order and first-order solutions.

The zeroth-order equations are defined by neglecting the scattering phase matrix terms $F^{\pm}(z)$ which multiplied by k_s, when k_s is very small.

Thus, after solving the radiative transfer equations, the total emission in the medium above the layer as zeroth-order solution can be simplified as,

$$T_p^{t0} = \left(T_s \cdot E_p^v + T_s \cdot E_p^v(1-E_p^s)L_p + T_g \cdot E_p^s \cdot L_p\right)\cdot \psi_p \tag{2}$$

where ψ_p, T_s, T_g are the power transmittivity at air-snow interface, snow temperature and ground temperature.

To seek the iterative first-order solution with the same contribution sources as zeroth-order, equations governing the first-order results are obtained by solving the source functions $F^{\pm}$ in which $T^{\pm}$ has been replaced by the zeroth-order solution $T_0^{\pm}$. Thus, we can derive the first-order solutions similar to the zeroth-order formulation,

$$T_p^{t1} = \left(T_s \cdot E_p^v + T_s \cdot E_p^v(1-E_p^s)L_p + T_g \cdot E_p^s \cdot L_p + E_1\right)\cdot \psi_p \tag{3}$$

Where E_1 is the volume scattering due to the single scattering within snow pack and between snow pack and underground surface. Its formulation can be seen in [3].

III. SENSITIVITY TEST ON EFFECTS OF SCATTERING ORDER

The multi-scattering microwave emission model we used in this study showed agreement well with the field observation [7]. With the confirmed multi-scattering model, we can do the following sensitivity test of different scattering-order emission models, and do the parameterization on our multi-scattering emission model.

In order to develop snow depth (SWE) inversion algorithm, it is necessary to analyze the scattering-order effect on the snow emission and examine the effects of different emission components. The zeroth-order, first-order solutions can be solved using iterative techniques. They are easily applied to do inversion due to their simple analytical formulation, and they provide understanding and insight into the physics of the sources and mechanisms of scattering. In this section, we will do sensitivity analysis of these three scattering-order emission models: the zeroth-order, first-order and the multi-scattering models.

We do two kinds of sensitivity tests. The first is to compare emission of different scattering-order solution with albedo increasing at 0° and 55° incidence angle. The second is to see the difference of different scattering-order solution when the layer optical thickness varying.

We use the different grain size for the albedo variation, and keep snow density, depth and temperature constant during all the sensitivity tests. We not only give the comparison at different incident angle under the same ground surface, we also make the comparison with optical thickness varying while kept other parameters.

Figure 1 and figure 2 are the comparisons of three different scattering-order models with albedo varying at 0° and 55° incidence angles. We can see that, at 0° incidence angle, the emission both at V-pol and H-pol are the same for these different scattering-order models. While the albedo is small less than 0.2, there are nearly no difference between these three solutions, i.e. the multi-scattering is really very small, so we can use zeroth-order or first-order solution for the layer emission calculation under such condition. With albedo increasing, the difference between these three solutions became larger due to the multi-scattering. And we also can see that the emission difference of the scattering-order models at 55° incidence angle is larger than that at 0° incidence angle.

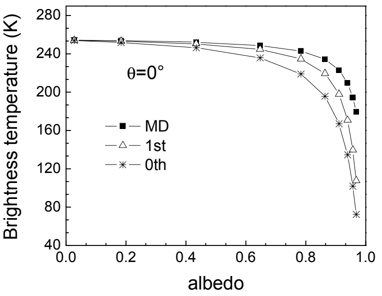

Fig 1. Emission comparison for different scattering-order solutions with different incident angle at V-pol at the incidence angle of 0°

* MD – the Matrix Doubling method, 1st – the first-order solutions, 0th – the zeroth-order solutions

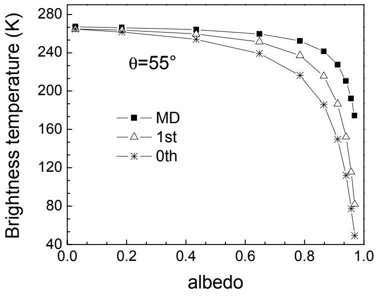

Fig 2. Emission comparison for different scattering-order solutions with different incident angle at V-pol at the incidence angle of 55°

Figure 3 showed the emission comparison of different scattering-orders with the same underground surface properties for V polarization at 55° incident angle. With the optical thickness becoming large, the layer emisson from the zeroth-order and the first-order solutions decreased rapidly, while the trend of the multi-scattering caused is relatively slowly.

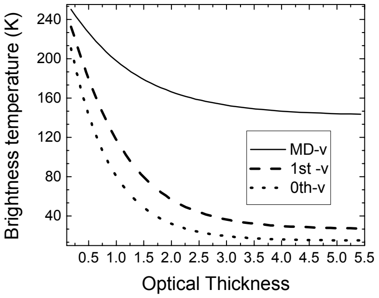

Fig 3 Emission comparison for different scattering-order solutions with optical thickness at V-pol at the incidence angle of 55°

In addition, we can see that these three different scattering-order solutions all saturated when the optical thickness is larger than 3.0. That means at this thickness level, the microwave couldn't penetrate the surface covered with snow. According to the sensitivity test, it can be seen that multi-scattering couldn't be ignored with large albedo and large optical thickness for an inhomogenous layer when we develop an inversion technique for snow depth.

III. PARAMETRIZED MODEL

In the above section, we see that the zeroth-order and first-order couldn't predict emission very well under the condition of large albedo and optical thickness. And the multi-scattering microwave emission model we used in this study showed agreement well with the field observation [7]. Obviously, we couldn't use them to do retrieval algorithm directly. Since this dense media emission model with multi-scattering is relatively complex and costs lots of computational time, it's necessary to develop a simple parameterized model. We are trying to improve the current AMSR-E semi-empirical snow depth retrieval algorithm. Then this parameterized model is at 55° incidence angle. First, we established the simulated database covered the most possible natural snow properties and underground character with the above emission model. We expect we could parameterize this numerical model keeping the formulation as that of zeroth-order solution, which consist of the snowpack emission, underground emission, and their interaction term. Then we can use this simple parameterized model to develop an inversion model to estimate snow depth.

From the above section, the zeroth-order emissivity can be expressed as

$$E_p^t = (E_p^v + E_p^v(1-E_p^s)L_p + E_p^s \cdot L_p) \cdot \psi_p \qquad (4)$$

where E_p^v, E_p^s is the snowpack and underground emissivity. While the parameterized model will be expressed as

$$E^t_{mp} = (E^v_{mp} + E^{vs}_{mp} + E^s_p \cdot L_p) \cdot \psi_p \tag{5}$$

In the above emission, the snow pack emission with multi-scattering can be approximated as

$$E^v_{mp} \approx E^v_p \cdot Cf^v_p \tag{6}$$

Where the correction factor Cf^v_p can be expressed as

$$\begin{aligned} Cf^v_p = \exp(a + b \cdot \log(L_p) + c \cdot \omega + d \cdot w^2 \cdot \log(L_p) \\ + e \cdot w^3 \cdot \log(L_p)) \end{aligned} \tag{7}$$

In addition, the sum of interaction and ground emission can be parameterized as

$$E^{sv}_{mp} + E^s_p \cdot L_p \approx E^s_p \cdot Cf^{svs}_p \tag{8}$$

Where the correction factor Cf^{svs}_p can be expressed as

$$\begin{aligned} Cf^{svs}_p = L_p \cdot \exp(a + b \cdot \log(L_p) + c \cdot \omega \\ + d \cdot w \cdot \log(L_p) + e \cdot (w \cdot \log(L_p))^2) \end{aligned} \tag{9}$$

where ω is albedo, L_p is attenuation factor in the snow pack. The regression coefficients *a*, *b*, *c*, *d*, *e* in the eq. (7), (9) are related to frequencies and incidence angles. All the above simplified models agreed well with the theoretical model when snow depth is larger than penetration depth.

The table1 is the RMSE (Standard error) of simplified model with theoretical DMRT.

Table1. The RMSE of simplified model VS. DMRT

RMSE	10.7 GHz	18.7G Hz	36.5G Hz
Snowpack emission	0.0004	0.0023	0.004
Sum of interaction term and ground emission (V)	0.004	0.008	0.001
Sum of interaction term and ground emission (H)	0.0084	0.015	0.016

From the above comparison, as can be seen that this parameterized model approach the theoretical model fairly well at 55° incident angle. The RMSE at 10.7 GHz are the smallest both for snow pack and the sum of interaction term and ground emission, because the multi-scattering at lower frequencies is not higher as that at high frequencies. The multi-scattering plays more important role in total snow emission prediction with frequencies increasing. The parameterized model provides us an insight to develop inversion techniques.

IV. CONCLUSIONS

From the sensitivity test of different scattering-order microwave emission models, it can be seen that we must take into account the multi-scattering in the snow layer emission with large albedo esp. for large grain size. Finally, we develop a parameterized model to simplify the complex but accurate emission model. We still need to improve this parameterized model with any frequencies and any incidence angles. In future, we'll expect to develop a snow depth inversion algorithm with the parameterized model.

V. ACKNOWLEDGEMENTS

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VI. REFERENCES

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