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## Dynamic environmental policy with strategic firms: prices versus quantities

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### Abstract

Environmental regulators often have imperfect information about regulated firms' abatement costs. In this paper we compare taxes and emissions permits in a dynamic setting in which firms behave strategically. The regulator updates policy over time based upon previous aggregate industry performance, assuming that firms are not strategic. We find that strategic firms facing an emissions tax have an incentive to overabate in order to obtain a lower tax in the future. Firms that trade emissions permits have a strategic incentive to reveal an artificially high permit price to obtain more permits in the future. Whether permits or taxes are preferred from a welfare standpoint depends upon how permit prices are determined. Taxes generate higher welfare when the low-cost firm sets the permit price but permits generate higher welfare when the high-cost firm sets the permit price.

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### 1. Introduction

Seldom does an environmental regulator know as much about regulated firms' abatement costs as do the firms themselves. This informational asymmetry constitutes one of the great difficulties of policymaking, for it implies that firms may have both the opportunity and the incentive to exploit their advantage to undermine the intended goals of a well-meaning regulator.

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Even if firms do not take advantage of their knowledge, a regulator's uncertainty about industry costs can lead to inefficient policies. In an early paper devoted to this matter, Weitzman [12] investigated whether, in the presence of uncertainty, price or quantity controls are preferable. Weitzman showed that a quantity instrument (resp. a price instrument) is preferred if the marginal benefit curve is steeper than (resp. less steep than) marginal costs.<sup>1</sup> Weitzman's model is static, so firms cannot strategically manipulate the regulator's beliefs about costs via their abatement strategy. Kwerel [7] developed a hybrid price–quantity instrument that induces competitive firms to reveal their true costs to the regulator, who then implements a first-best outcome. More recently, Kaplow and Shavell [6] claimed that the first-best outcome can also be achieved with a non-linear tax set equal to the (non-linear) pollution damage function. Whether schemes that are more involved than simple linear tax or quantity instruments could be implemented in practice is an open question.

While these models are static, the world is not and it seems reasonable to ask whether informational asymmetry plays a different role in dynamic models. In recent years, a number of articles have appeared that take up this question. Following Weitzman's lead, Newell and Pizer [8] and Hoel and Karp [5] investigated the effect of stock pollutants on the price–quantity question. As with the static models, they found that taxes are preferred to quotas when the slope of the marginal abatement costs is large relative to the slope of the marginal damages. In addition, taxes dominate if the discount rate is high or the pollution stock has a high decay rate. Their results are robust to changes in parameter values but conditional on the assumption of quadratic functions and additive uncertainty. Baldursson and von der Fehr [3] found that, in a dynamic and uncertain model, any irreversibility in abatement decisions can affect policy choice generally as well as the price–quantity comparison.<sup>2</sup> In a paper that extends Kwerel's model to a dynamic setting, Benford [4] showed that when firms are perfectly competitive (that is, non-strategic), the natural extension of Kwerel's scheme can induce the optimal trajectory of abatement over time.

To our knowledge, in all of the work that compares price and quantity instruments in a dynamic setting, it is assumed that firms are non-strategic price takers. It is the regulator who adopts sophisticated dynamic policy rules in order to induce a pliant, non-strategic polluting sector to achieve socially desirable outcomes. Though it has produced many important insights, it would appear that this approach runs counter to Weitzman's assumption about who holds the advantage in the interaction between regulator and polluters. If polluting firms have the informational advantage and are large enough to realize that their actions can influence regulatory outcomes, it would seem natural to study a dynamic policy setting in which they are also more sophisticated than the regulator.

In the present paper we set out to do just that. First, we develop a dynamic two-period model of environmental regulation in which two regulated firms are able to manipulate the regulator. We then extend the model to  $T$  periods, where  $T$  is possibly infinite. The regulator knows the function describing abatement benefits, but faces uncertainty regarding the firms' abatement costs. Two policy instruments are compared: emissions taxes and emissions permits. In each case, the

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<sup>1</sup>Other studies in the Weitzman tradition include Adar and Griffin [1], Roberts and Spence [9], and Stavins [11].

<sup>2</sup>See also, for example, Rubio and Escriche [10], who deal with international carbon-trading issues. Typical findings include the fact that the optimal time path of the carbon tax is extremely sensitive to the pollution accumulation process.

regulator sets the policy in the first period so that marginal benefits equal her expectation of industry marginal abatement costs. In each subsequent period, the regulator adjusts the policy based on observed price and quantity in the preceding period.

We assume that the regulator follows a mechanical but natural rule for updating the policy from one period to the next. Specifically, she believes that the firms are non-strategic and will behave so that marginal cost equals the tax (in the case of emissions taxes) or the permit price (in the case of emissions permits). Having observed the firms' behavior in period  $t$ , the regulator derives a new estimate of marginal costs and sets the policy in period  $t + 1$  so that marginal benefits equal the new expected marginal cost function. A key assumption is that the firms know this rule, and that their behavior in each period optimally anticipates the effect of their action on the policy in the following period. The regulator, then, is at a disadvantage in two respects. First, she is uncertain about the firm's abatement cost functions. Second, while the firms know the regulator's dynamic rule, it is assumed that the regulator does not anticipate the firms' manipulation of the rule.

Though the regulator in this model is relatively unsophisticated compared to the firms, there are several situations in which this may be a close reflection of reality. If a given policy includes a grandfathering provision, polluters might be able to affect their treatment under the anticipated regulatory policy in the future by modifying their behavior today. One could also think of our regulator's adjustment rule as a legislative mandate or statute that firms in the affected industry can influence through their actions. However one chooses to interpret the regulator's rule, it should be kept in mind that there is an important difference between our model, and that of Kwerel [7] and others in the mechanism-design literature. In mechanism design, the regulator implements a mechanism for which it is optimal for firms to reveal information truthfully. In our model, firms must deliberately alter their behavior to send an incorrect signal regarding their costs. In this way our model resembles that of Weitzman [13], who examined the "ratchet effect" and described optimal policy for an enterprise whose future performance targets are based on current performance. In a paper that is perhaps most similar to ours, Andersson [2] applied the ratchet idea to pollution control. He considered only a permit scheme, however, while we compare permits and taxes, and his firms were able to collude in reaching their trading decisions while ours cannot.

We derive two main sets of results. The first concerns the level of emissions in the tax and permits cases. On one hand, firms facing an emissions tax have a strategic incentive to overabate, pretending that their costs are low. The regulator, then, believing that the firms have low abatement costs, sets a relatively low tax. But overabatement is expensive, and the dynamic problem facing the firms requires balancing the desire to appear to have low costs against the desire to minimize actual abatement costs. On the other hand, firms facing a permit market have a strategic incentive to reveal a high permit price. The regulator, believing then that the firms have high abatement costs, issues more permits in the next period.

In the two-period model with emissions taxes, we show that firms overabate in the first period and underabate in the second period relative to non-strategic firms. In the steady-state equilibrium of the infinite-horizon model, firms overabate and face lower taxes relative to non-strategic firms. The results for the permit case are both more complicated and less general. The complexity arises because firms have different incentives on how to manipulate the permit price depending upon whether they are a buyer or a seller of permits. All firms have a strategic incentive to set high permit prices because this leads the regulator to believe that abatement costs are high and to

allocate more permits in the next period. However, a firm that buys permits wants the price to be low in order to reduce the cost of permit purchases. Whether the permit price is set above or below marginal abatement cost depends on whether the high-cost firm (likely buyer) or low-cost firm (likely seller) has more influence in determining the permit price. When the high-cost firm sets the permit price and the price effect dominates the strategic effect, the permit price will be set below marginal abatement cost. Otherwise, the permit price will be set above marginal abatement cost. In a steady-state equilibrium, if the permit price is set above (below) marginal abatement cost, the number of permits allocated by the regulator will be more (less) than the efficient level and firms will abate less (more) than the efficient amount. To demonstrate the range of possible outcomes, we solve two polar opposite cases: one in which the low-cost firm is a monopoly seller of permits and sets the permit price, and one in which the high-cost firm is a monopsony buyer of permits and sets the permit price.

The second set of results, comparing welfare under permits and taxes, is derived from a numerical simulation of the multi-period model. Even with linear marginal benefit and marginal cost functions, the dynamic model becomes unwieldy, with fourth-order polynomials describing the objective function in the first period of a two-period model. Because analytic solutions are unavailable, and because we want to go beyond comparative statics to compare welfare levels, we conduct a series of numerical exercises aimed at comparing the two policy instruments. The numerical results highlight the importance of the way in which the market permit price is determined in choosing between the two policy instruments. We find that when the low-cost firm sets the permit price, taxes outperform permits in terms of welfare. On the other hand, when the high-cost firm sets the permit price, permits outperform taxes. These results hold regardless of whether marginal cost is steeper or flatter than marginal benefit.

## 2. The two-period model

There are two firms, one with high abatement costs and the other with low abatement costs, indexed by  $j = h, l$ . There are two time periods, indexed by  $t = 1, 2$ . Let  $\delta$  be the discount factor between periods. Initial emissions (without any costly abatement activity) by firm  $j$  in period  $t$  are  $e_t^j$ . Let abatement by firm  $j$  in period  $t$  be  $q_t^j$ . The abatement cost function for firm  $j$  is given by  $C^j(q_t^j, \theta)$ . Here,  $\theta$  is a realization of a random variable  $\Theta$ , which is known to the firms but not known by the regulator. We assume that the marginal abatement cost function for each firm is positive and increasing in abatement ( $C_q^j(q_t^j, \theta) > 0$  and  $C_{qq}^j(q_t^j, \theta) > 0$ ) and that  $C^j(0, \theta) = 0$ . We assume that increases in  $\theta$  result in higher total and marginal abatement cost at all levels of abatement:  $C_\theta^j(q_t^j, \theta) > 0$  and  $C_{q\theta}^j(q_t^j, \theta) > 0$ . The aggregate abatement cost function  $C(q_t, \theta)$  is the minimum cost of achieving aggregate abatement in period  $t$ , where aggregate abatement is  $q_t = q_t^h + q_t^l$ . Let  $C_q(q_t, \theta)$  represent the marginal aggregate abatement cost function, assumed to be continuous. Define  $E[C_q(q_t, \Theta)]$  as the *expected* marginal abatement cost function when information about the realization of the random variable  $\Theta$  is unknown. Benefits of abatement in period  $t$  are  $B(q_t)$ . We assume that the marginal benefits of abatement are positive but declining in aggregate abatement:  $B'(q_t) > 0$  and  $B''(q_t) < 0$ . To ensure an interior optimum, we assume that  $C_q(0, \theta) < B'(0)$  for all  $\theta$  in the support of  $\Theta$ .

Prior to the first period, the regulator chooses a type of policy, either emissions taxes or marketable emissions permits. In period 1, she sets the level of emission taxes or the number of permits issued to each firm. Firms then choose period 1 abatement (and emissions trading). The regulator observes each firm's abatement level and, in the case of marketable emissions permits, the price of permits. She then updates her belief about  $\theta$  and, based upon the firms' behavior, infers that the value of the random variable  $\Theta$  is  $\theta^R$ . In the second period, the regulator again sets the level of emission taxes or the number of permits issued to each firm, this time using information gathered from observing first-period emissions and prices. Firms then choose period 2 abatement (and emissions trading).

The goal of each firm is to minimize the present value of costs (abatement plus regulatory costs). Firms are strategic in that they take account of how first-period actions may influence future regulatory policy.

The regulator's objective is to maximize the expected present value of net social benefits (i.e., minimize the sum of damages from pollution and abatement cost). In our model, the regulator is not strategic in the same way that firms are. In each period, she sets marginal benefit equal to expected marginal cost and sets policy accordingly. The regulator uses a non-strategic updating of beliefs in period 2 that fails to account for the firms' strategic behavior.

### 2.1. Emissions taxes

In the first period, the regulator chooses an emissions tax,  $p_1$ , such that marginal benefits equal expected aggregate marginal costs:

$$p_1 = B'(q_1) = E[C_q(q_1, \Theta)].$$

In response to  $p_1$ , each firm chooses an abatement level,  $q_1^j, j \in \{h, l\}$ . The regulator observes and uses this information to update her belief in a non-strategic fashion. She believes that firms set abatement so that the emissions tax equals marginal abatement cost,  $p_1 = C_q^j(q_1^j, \theta)$ , and therefore that  $p_1 = C_q(q_1, \theta)$ . Following this belief, the regulator infers that the realization of  $\Theta$  is  $\theta^R$ . The regulator then sets the second-period emissions tax,  $p_2 = g(q_1 | p_1)$ , such that

$$B'(q_2) = C_q(q_2, \theta^R) = g(q_1 | p_1).$$

**Proposition 1.** *Greater abatement in the first period results in lower emissions taxes in the second period:  $g'(q_1 | p_1) < 0$ .*

**Proof.** We can expand  $g'(q_1 | p_1)$  as follows:

$$g'(q_1 | p_1) = \frac{dp_2}{dq_1} = \frac{dp_2}{dq_2} \frac{dq_2}{d\theta^R} \frac{d\theta^R}{dq_1}.$$

In the first period, the regulator believes that abatement is chosen such that  $C_q(q_1, \theta^R) = p_1$ . Totally differentiating this equation with respect to  $q_1$  and  $\theta^R$  yields

$$C_{qq}(q_1, \theta^R) dq_1 + C_{q\theta}(q_1, \theta^R) d\theta^R = 0,$$

which becomes, after rearranging,

$$\frac{d\theta^R}{dq_1} = -\frac{C_{qq}(q_1, \theta^R)}{C_{q\theta}(q_1, \theta^R)} < 0.$$

High values of  $q_1$  are a signal to the regulator of low cost (i.e., a low value of  $\theta^R$ ).

In the second period, the regulator wants abatement levels to satisfy  $B'(q_2) = C_q(q_2, \theta^R)$ . Totally differentiating with respect to  $q_2$  and  $\theta^R$  yields

$$B''(q_2) dq_2 = C_{qq}(q_2, \theta^R) dq_2 + C_{q\theta}(q_2, \theta^R) d\theta^R,$$

which becomes, after rearranging,

$$\frac{dq_2}{d\theta^R} = \frac{C_{q\theta}(q_2, \theta^R)}{B''(q_2) - C_{qq}(q_2, \theta^R)} < 0.$$

When the regulator expects costs to be low (a low value of  $\theta^R$ ), marginal cost equals marginal benefit at high levels of abatement.

In the second period the regulator will choose to set the emissions tax equal to marginal benefits:  $p_2 = B'(q_2)$ . Differentiating this expression with respect to  $q_2$  yields

$$\frac{dp_2}{dq_2} = B''(q_2) < 0.$$

Therefore, we have  $g'(q_1 | p_1) < 0$ . □

Because firms are strategic, they will take account of how the regulator responds to the first-period choice of abatement. The two-period total cost for firm  $j$  is

$$TC^j = p_1(e_1^j - q_1^j) + C^j(q_1^j, \theta) + \delta\{g(q_1^j + q_1^{-j} | p_1)(e_2^j - q_2^j) + C^j(q_2^j, \theta)\}, \tag{1}$$

where  $q_1^{-j}$  denotes the vector  $q_1$  with the  $j$ th component removed. Note that there is limited strategic interaction between firms in this problem. When the second period arrives, the tax  $p_2$  has been set and both firms act as price takers. Therefore, in period 2 a firm does not care what its rival chooses to abate. In period 1, however, firms wish to manipulate the regulator's belief about  $\theta$  to obtain more favorable tax treatment in the second period. The regulator bases  $p_2$  upon aggregate observed first-period abatement. The firms choose  $q_1^j$  simultaneously and in the Nash equilibrium of this game each fails to account for the benefit that its own overabatement confers on the other firm. Thus, they do not achieve the collusive abatement levels. Importantly, neither do they seek to manipulate the other firm's second-period behavior through their choice of  $q_1^j$ . Therefore, minimizing (1) with respect to  $q_1^j$  and  $q_2^j$  yields unique subgame-perfect equilibrium strategies.

Letting  $q_t^{j*}$  denote optimal abatement for firm  $j$  in period  $t$ , the first-order conditions for an interior solution to minimize total cost are

$$C_q^j(q_1^{j*}, \theta) - p_1 + \delta g'(q_1^{j*} + q_1^{-j} | p_1)(e_2^j - q_2^{j*}) = 0$$

and

$$C_q^j(q_2^j, \theta) - p_2 = 0,$$

for  $j \in \{h, l\}$ . In contrast to the conditions characterizing subgame-perfect equilibrium, non-strategic firms will set marginal abatement costs equal to the emissions tax:

$$C_q^j(\hat{q}_1^j, \theta) - p_1 = 0$$

and

$$C_q^j(\hat{q}_2^j, \theta) - p_2 = 0,$$

where  $\hat{q}_t^j$  denotes the equilibrium abatement levels chosen by non-strategic firms. Note that with non-strategic firms  $p_2$  will be the efficient tax level because the regulator can correctly infer marginal costs by observing abatement in the first period.

**Proposition 2.** *In a subgame-perfect equilibrium of the game with emissions taxes, firms will overabate in the first period relative to equilibrium with non-strategic firms. In addition, firms will face lower taxes in the second period and underabate in the second period relative to the efficient outcome.*

**Proof.** In equilibrium,  $q_1^{j*}$  satisfies  $C_q^j(q_1^{j*}, \theta) = p_1 - \delta g'(q_1^{j*} + q_1^{-j} | p_1)(e_2^j - q_2^{j*})$ . Because  $g'(q_1 | p_1) < 0$ , we know that  $p_1 - \delta g'(q_1^{j*} | p_1)(e_2^j - q_2^{j*}) > p_1$ . This means that  $C_q^j(q_1^{j*}, \theta) > C_q^j(\hat{q}_1^j, \theta)$ . Therefore, because  $C_{qq}^j(q_1^{j*}, \theta) > 0$  for both  $j$ , it must be true that  $q_1^{j*} > \hat{q}_1^j$ . From Proposition 1, higher first-period abatement yields a lower second-period emissions tax. Because the tax is lower in the second period, abatement will also be lower than it would have been with non-strategic firms.  $\square$

## 2.2. Marketable emissions permits

Let  $a_t$  equal the total number of marketable emissions permits allocated by the regulator in period  $t$ , and let  $a_t^j$  equal the number of permits allocated to firm  $j$  in period  $t$ , with  $a_t^h + a_t^l = a_t$ . We assume that the regulator follows a rule for allocating the permits between the two firms; the rule is common knowledge. Firms are not allowed either to borrow permits in the first period or to bank them for future use. In each period, total emissions must not exceed total marketable emissions permits:

$$e_t^h + e_t^l - q_t^h - q_t^l \leq a_t^h + a_t^l,$$

or  $e_t - q_t \leq a_t$ . In equilibrium, these expressions will hold with equality; there will be no unused permits.

In the first period, the regulator sets  $a_1$  so that marginal benefits equal expected marginal costs:  $B'(e_1 - a_1) = E[C_q((e_1 - a_1), \Theta)]$ . Firms then choose their individual abatement levels and trade permits. The regulator observes the market permit price,  $p_{a_1}$ . The regulator believes that firms trade so that the market permit price equals the marginal cost of abatement, and hence that

$p_{a_1} = C_q((e_1 - a_1), \theta)$ . From this belief, the regulator infers that the realization of  $\theta$  is  $\theta^R$ . She then sets the second-period total allocation of permits,  $a_2 = h(p_{a_1} | a_1)$ , so that  $B'(e_2 - a_2) = C_q((e_2 - a_2), \theta^R)$ .

**Proposition 3.** *A higher permit price in the first period results in more permits being allotted in the second period:  $h'(p_{a_1} | a_1) > 0$ .*

**Proof.** We may expand  $h'(p_{a_1} | a_1)$  as follows:

$$h'(p_{a_1} | a_1) = \frac{da_2}{dp_{a_1}} = \frac{da_2}{d\theta^R} \frac{d\theta^R}{dp_{a_1}}.$$

In the first period, the regulator believes that abatement is chosen such that  $C_q((e_1 - a_1), \theta^R) = p_{a_1}$ . Totally differentiating this equation with respect to  $p_{a_1}$  and  $\theta^R$  yields

$$dp_{a_1} = C_{q\theta}((e_1 - a_1), \theta^R) d\theta^R,$$

which in turn leads to

$$\frac{d\theta^R}{dp_{a_1}} = \frac{1}{C_{q\theta}(q_1, \theta^R)} > 0.$$

High prices for permits in period 1 are a signal to the regulator of high cost (i.e., a high value of  $\theta^R$ ).

In the second period, the regulator wants abatement levels to satisfy  $B'(e_2 - a_2) = C_q((e_2 - a_2), \theta^R)$ . Totally differentiating  $B'(e_2 - a_2) = C_q((e_2 - a_2), \theta^R)$  with respect to  $a_2$  and  $\theta^R$  yields

$$-B''(e_2 - a_2) da_2 = -C_{qq}((e_2 - a_2), \theta^R) da_2 + C_{q\theta}((e_2 - a_2), \theta^R) d\theta^R,$$

which in turn leads to

$$\frac{da_2}{d\theta^R} = \frac{C_{q\theta}(q_2, \theta^R)}{C_{qq}((e_2 - a_2), \theta^R) - B''(e_2 - a_2)} > 0.$$

When the regulator expects costs to be high (that is, when she observes a high value of  $\theta^R$ ), marginal cost equals marginal benefit at low levels of abatement or high levels of emissions, so the regulator allots a high number of permits. Therefore, we have  $h'(p_{a_1} | a_1) > 0$ .  $\square$

Proposition 3 shows that firms have a strategic incentive to set high prices in order to get the regulator to allocate more permits in the next period. Note that the strategic effect with marketable emissions permits works in the opposite direction of the effect in the case with emissions taxes, as shown in Proposition 1. To receive more lenient regulatory treatment with marketable emissions permits, the firms try to convince the regulator that abatement costs are high. When the regulator believes that abatement costs are high, she will allocate more permits in the following period. In contrast, for the case with emissions taxes, firms try to convince the regulator that abatement costs are low, leading her to set low taxes in the following period.

The analysis of equilibrium in the case of marketable emissions permits is complicated by the fact that firms interact with each other in the permit market, as well as with the regulator. Trading in the permit market gives rise to another set of incentives about where to set the price. A firm that sells permits prefers high permit prices while a firm that buys permits would prefer a low permit price. Further, the number of permits allocated in a period may affect the resulting equilibrium price and the division of rents between firms. Thus, it is not clear how the equilibrium outcome and the efficient outcome compare.

In the permit market the two firms are engaged in a bilateral monopoly game. There is, of course, no unique solution to a bilateral monopoly. We consider two extreme cases. In one case the low-cost firm sets the price at which trades may occur.<sup>3</sup> Because the low-cost firm is typically a seller of permits, this case is akin to monopoly. In the other case, the high-cost firm sets the permit price. Because the high-cost firm is typically a buyer of permits, this case is akin to monopsony. In actual bargaining situations, typically both firms would have a degree of bargaining power and the price would reflect the bargaining power of each firm.

In each period, the firm setting the price is the price leader, denoted with superscript L, and the other firm is the follower, denoted with superscript F. In the second period, the problem facing the follower, given an allocation of permits  $a_2^F$  and facing permit price  $p_2$ , is to maximize the net permit revenue minus abatement cost:

$$\max_{q_2^F} \{p_2(a_2^F + q_2^F - e_2^F) - C^F(q_2^F, \theta)\}.$$

The solution to this problem involves setting the permit price equal to marginal abatement cost. Let  $q_t^F(p_t)$  be the abatement level that equates marginal abatement cost and price for given price  $p_t$  in period  $t$ . Because marginal abatement cost is increasing,  $\partial q_2^F / \partial p_2 > 0$ .

The leader takes account of the follower's reaction when setting the price. Using the fact that total emissions will equal total marketable permits, so that  $q_2^L = e_2 - a_2 - q_2^F(p_2)$ , the dynamic programming equation for the leader is:

$$V_2^L(a_2; \alpha) = \max_{p_2} \{p_2(e_2^F - q_2^F(p_2) - a_2^F) - C^L(e_2^F - q_2^F(p_2) - a_2^F, \theta)\},$$

where  $\alpha$  denotes the fraction of permits granted to the leader:  $a_t^L = \alpha a_t$  and  $a_t^F = (1 - \alpha)a_t$ . Rearranging the first-order conditions for this problem yields

$$(p_2 - C_q^L(q_2^L, \theta)) \frac{\partial q_2^F}{\partial p_2} = e_2^F - q_2^F(p_2) - a_2^F.$$

The right side of this equation represents the net purchases of the follower, which will be positive if the follower is a net buyer and negative if the follower is a net seller. The leader will set the price above (below) its marginal abatement cost when it sells to (buys from) the follower.

The regulator sets the second-period allocation of permits based on the period 1 price:  $a_2 = h(p_1 | a_1)$ . In period 1,  $p_1$  is taken as given by the follower so the follower cannot influence the

<sup>3</sup>As a reviewer has suggested, the firm with market power might be given the opportunity to extract all of the surplus. This would involve non-linear pricing and would allow the firm to ensure an efficient division of abatement. It would not, however, produce a single price upon which the regulator could base the next period's policy. For that reason we require the firm with market power to select a unique price.

period 2 allocation of permits. Therefore, the follower will again choose abatement so that marginal abatement cost equals price. Let  $q_1^F(p_1)$  be the abatement level that equates marginal abatement cost and price for a given price  $p_1$ . In period 1, the leader’s problem is

$$V_1^L(a_1^L, a_1^F) = \max_{p_1} \{p_1(e_1^F - q_1^F(p_1) - a_1^F) - C^L(e_1^F - q_1^F(p_1) - a_1^F, \theta) + \delta V_2^L(h(p_1 | a_1); \alpha)\}. \tag{2}$$

Rearranging the first-order conditions for this problem yields

$$(p_1 - C_q^L(q_1^L, \theta)) \frac{\partial q_1^F}{\partial p_1} = [e_1^F - q_1^F(p_1) - a_1^F] + \left[ \delta \left( \frac{\partial V_2^L(a_2; \alpha)}{\partial a_2} \right) \left( \frac{\partial h(p_1 | a_1)}{\partial p_1} \right) \right]. \tag{3}$$

The second term on the right side of this equation, which did not appear in the equivalent expression in the solution from the second period, is the strategic regulatory term. From Proposition 3 we know that  $\partial h(p_1 | a_1) / \partial p_1 > 0$ . Typically,  $\partial V_2^L(a_2; \alpha) / \partial a_2 > 0$ , which means that more permits are good for a firm’s bottom line. But this result need not always hold. For example, when a leader sells permits to a follower with very steep marginal abatement costs, the leader may be better off when obtaining fewer permits in the second period. With few permits, the equilibrium permit price may be quite high. If demand for permits is highly inelastic, the revenue effect may outweigh the direct effect from the firm having fewer permits, either to sell or to use. Because the signs of both bracketed terms on the right side of Eq. (3) are ambiguous, it is not possible to say for sure whether the leader will set price above or below marginal abatement cost in the first period. We explore this issue further using simulations in Section 4.

### 3. The $T$ -period model

In this section we analyze a case with  $T$  periods,  $t = 1, 2, \dots, T$ , where  $T$  is possibly infinite. We seek to determine the degree to which the main results of the two-period model continue to hold as the time horizon is extended. We retain all of the structure of the two-period model, except that the time horizon is lengthened.

#### 3.1. Emissions taxes

We assume that in each period the regulator sets the tax based on an assessment of  $\theta$ , now denoted  $\theta_t^R$ , which is based only upon the abatement quantity observed in the previous period:  $p_{t+1} = g(q_t | p_t)$ . Note that this is analogous to the two-period model, where  $p_2 = g(q_1 | p_1)$ . As in the two-period model, we continue to assume that neither the benefit function nor the cost functions change from period to period.

The problem facing the two firms is set up as a dynamic programming problem. In period  $T$ , firm  $j$ ’s problem is

$$V_T^j(p_T) = \max_{q_T^j} \{-p_T(e_T^j - q_T^j) - C^j(q_T^j, \theta)\}.$$

The solution requires setting emissions so that marginal abatement cost equals the emissions tax. Note that  $\partial V_T^j(p_T) / \partial p_T < 0$ .

Now consider the problem starting in period  $T - 1$ . This problem is identical to the problem facing the firm in period one of the two-period model. Using a dynamic programming formulation, we can write the problem facing the firm with two periods remaining as follows:

$$V_{T-1}^j(p_{T-1}) = \max_{q_{T-1}^j} \{-p_{T-1}(e_{T-1}^j - q_{T-1}^j) - C^j(q_{T-1}^j, \theta) + \delta V_T^j(p_T(q_{T-1}))\}. \quad (4)$$

Once again letting  $q_{T-1}^{j*}$  denote firm  $j$ 's optimal abatement level, the necessary condition for an interior solution to this problem is

$$p_{T-1} - C_q^j(q_{T-1}^{j*}, \theta) + \delta \left( \frac{\partial V_T^j(p_T(q_{T-1}))}{\partial p_T} \frac{\partial p_T}{\partial q_{T-1}^{j*}} \right) = 0.$$

The final term in this equation is the strategic term, which captures the value to the firm of influencing the emissions tax in the next period by choice of current abatement. This term is positive because  $\partial V_T^j(p_T)/\partial p_T < 0$ , as shown above, and  $\partial p_T/\partial q_{T-1}^{j*} < 0$  by Proposition 1. In equilibrium, each firm sets abatement so that marginal abatement cost exceeds the emissions tax. As in the first period of the two-period model, firms overabate relative to non-strategic firms in the penultimate period.

Extending the analysis to period  $T - 2$ , the problem facing the firm with three periods remaining is

$$V_{T-2}^j(p_{T-2}) = \max_{q_{T-2}^j} \{-p_{T-2}(e_{T-2}^j - q_{T-2}^j) - C^j(q_{T-2}^j, \theta) + \delta V_{T-1}^j(p_{T-1}(q_{T-2}))\}.$$

The necessary condition for an interior solution of this problem is

$$p_{T-2} - C_q^j(q_{T-2}^{j*}, \theta) + \delta \left( \frac{\partial V_{T-1}^j(p_{T-1})}{\partial p_{T-1}} \frac{\partial p_{T-1}}{\partial q_{T-2}^{j*}} \right) = 0. \quad (5)$$

Except for the difference in time subscripts, this condition is identical to that shown for the problem beginning in period  $T - 1$ . The strategic term is again positive because  $\partial V_{T-1}^j(p_{T-1})/\partial p_{T-1} < 0$ , as can be seen by inspecting Eq. (4), and because  $\partial p_{T-1}/\partial q_{T-2}^{j*} < 0$ . The latter inequality holds because the regulator sets  $p_{T-1}$  based on her belief about costs that are fixed by abatement in period  $T - 2$ . Higher abatement leads the regulator to believe that costs are lower and therefore to set a lower emissions tax in the next period. In an equilibrium in period  $T - 2$ , each firm sets its level of abatement so that its marginal abatement cost exceeds the emissions tax, just as in period  $T - 1$ .

In general, at the start of time period  $t$ , the problem facing a firm is

$$V_t^j(p_t) = \max_{q_t^j} \{-p_t(e_t^j - q_t^j) - C^j(q_t^j, \theta) + \delta V_{t+1}^j(p_{t+1}(q_t))\}.$$

The necessary condition for an interior solution is identical to Eq. (5), again except for the time subscripts. As a consequence, at any time  $t < T$ , each firm sets its emissions so that its marginal abatement cost exceeds the emissions tax. Because each firm does this, it follows that the aggregate marginal cost exceeds the emissions tax:  $C_q(q_t^j, \theta) > p_t$ . We can use this result to show

that in a steady state, where prices and emissions do not change from period to period, firms will abate more than is efficient.

**Proposition 4.** *In a steady-state equilibrium with emissions taxes, firms abate more than is efficient.*

**Proof.** For a given value of  $\theta$ , define  $q^*$  as the efficient level of abatement (for which  $B'(q^*) = C_q(q^*, \theta)$ ) and define  $p^*$  as the emissions tax that would induce non-strategic firms to set the efficient level of abatement, ( $p^* = B'(q^*) = C_q(q^*, \theta)$ ). Define  $q^s$  as the steady-state level of abatement and  $p^s$  as the steady-state emissions tax according to the regulator’s updating rule. An argument similar to that following equation (5) ensures that  $C_q(q^s, \theta) > p^s$ . The regulator sets  $p^s$  so that  $p^s = B'(q^s) = C_q(q^s, \theta^R)$ , where  $\theta^R$  is the regulator’s steady-state belief about  $\theta$ . Therefore,  $C_q(q^s, \theta) > B'(q^s)$ . Finally, because  $C_{qq} > 0$  and  $B'' < 0$ , we know that  $q^s > q^*$ .  $\square$

The intuition for Proposition 4 is illustrated in Fig. 1. Firms abate at  $q^s > q^*$  because doing so makes the regulator believe that marginal abatement costs are  $C_q(q, \theta^R)$ , leading the regulator to set the emissions tax at  $p^s < p^*$ . The losses incurred by the firms due to overabatement are more than recovered through lower future taxes.

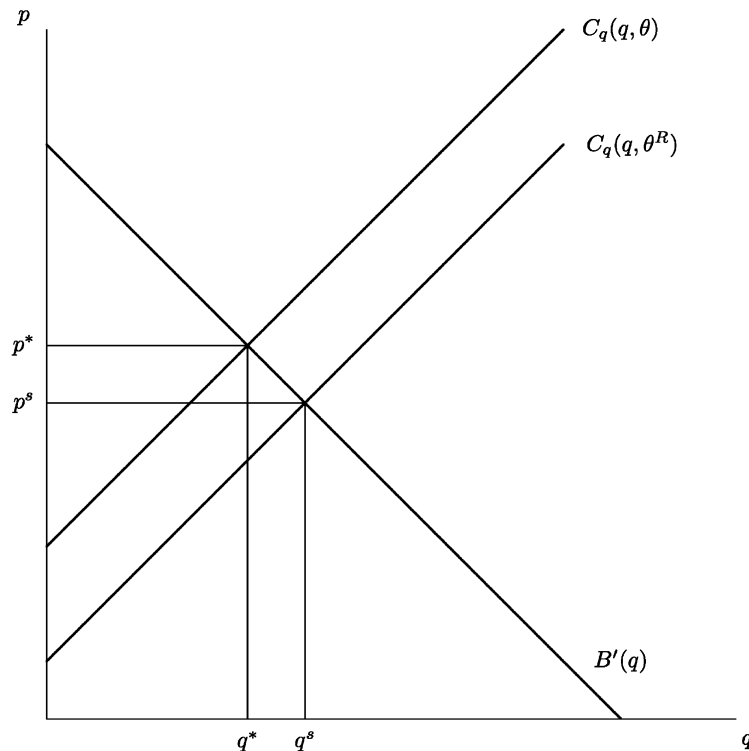


Fig. 1. Steady-state abatement, emissions taxes.

### 3.2. Marketable emissions permits

As was true for emissions taxes, the  $T$ -period model with marketable emissions permits differs from the two-period model only in the longer time horizon. The regulator sets the number of permits in period  $t$  based on  $\theta_t^R$ , her belief about  $\theta$ , which is in turn based on the permit price and the number of permits allocated in the previous period:  $a_t = h(p_{t-1} | a_{t-1})$ . We analyze the  $T$ -period model with marketable emissions permits using dynamic programming.

In period  $t$ , the follower faces a price set by the leader. Because the regulator responds only to the price and not the volume of trade, the follower's best response is to set marginal abatement cost equal to price. Let  $q_t^F(p_t)$  be the abatement level that equates marginal abatement cost and price for a given price  $p_t$ . The dynamic programming equation for the leader in period  $t$  is

$$V_t^L(a_t; \alpha) = \max_{p_t} \{p_t(e_t^F - q_t^F(p_t) - a_t^F) - C^L(e_t^F - q_t^F(p_t) - a_t^F, \theta) + \delta V_{t+1}^L(h(p_t | a_t); \alpha)\}.$$

This equation is virtually identical to Eq. (2) representing the choice for the leader in the first period of the two-period model. Rearranging the first-order conditions for this problem yields an equation similar to (3):

$$(p_t - c_q^L(q_t^L, \theta)) \frac{\partial q_t^F}{\partial p_t} = [e_t^F - q_t^F(p_t) - a_t^F] + \left[ \delta \left( \frac{\partial V_{t+1}^L(a_{t+1}; \alpha)}{\partial a_{t+1}} \right) \left( \frac{\partial h(p_t | a_t)}{\partial p_t} \right) \right].$$

As in the two-period model, the right side may be either positive or negative so that the emissions price is set either higher or lower than marginal abatement cost. In a steady-state equilibrium, if price is set above marginal abatement cost, then more than the efficient number of permits will be distributed and too little abatement will occur relative to the efficient amount. On the other hand, if price is set below marginal abatement cost, then too few permits will be distributed and more than the efficient amount of pollution will be abated.

## 4. Numerical optimization and results

The difficulty of this problem precludes us from comparing analytically the welfare outcomes of taxes and permits over a long time horizon. Firms fold the reaction of the regulator into their objective functions. Even in a two-period model with a quadratic objective function (linear marginal cost and marginal benefit), solving for equilibrium involves fourth-order polynomial expressions. We therefore turn to numerical optimization in order to compare optimal firm behavior, regulator response, and welfare over a  $T$ -period horizon. In particular, we specify cost and benefit functions and employ the technique of iterating on the value function, simultaneously obtaining optimal firm behavior as the solution to the dynamic game in each period. In this section we describe this approach in more detail for both the taxes and permits case, and then turn to the results of this numerical exercise.

### 4.1. Optimization methods

We specify a model with linear marginal benefits and linear marginal costs, consistent with the general model defined in section two, to compare welfare and policy outcomes under an emissions

tax and marketable emissions permits. Let the marginal benefit of abatement be given by

$$MB(q_t) = w + vq_t,$$

where  $w > 0$  and  $v < 0$  are constants. The marginal cost of pollution abatement for firm  $j$  ( $j \in \{h, l\}$  for the tax case and  $j \in \{L, F\}$  in the permits case) is

$$MC^j(q_t^j) = (d + \theta) + b^j q_t^j,$$

where  $w > (d + \theta) \geq 0$  for all values of  $\theta \in \Theta$ , and  $b^j > 0$ . The regulator's expected marginal cost is the horizontal sum of the expectation of the two firms' marginal costs:

$$E[MC_t] = d + E(\Theta) + Bq_t,$$

where  $B = b^h b^l / (b^h + b^l)$  is the slope of the aggregate marginal cost curve. In this model the regulator knows the slope of the aggregate marginal cost function but may have an incorrect assessment of the intercept.

In the calculations that follow, we explore several cases with different parameter values. The parameter values that do not change from one experiment to the next are:  $d = 10$ ,  $w = 50$ ,  $b^h = 3$ ,  $b^l = 1.5$ , and  $\alpha = 0.5$ .<sup>4</sup> We vary the parameters  $\delta$ ,  $\theta$ ,  $v$ , and  $e$  (the total unregulated emissions, where  $e^j = e^{-j} = e/2$ ) in order to explore their effects on the regulator's policies, optimal firm response, and overall welfare of using either a tax or a tradeable permit system. In the permits case, we explore two regimes: (1) the leader is the low-cost firm ( $b^L = 1.5$ ,  $b^F = 3$ ); and (2) the leader is the high-cost firm ( $b^L = 3$ ,  $b^F = 1.5$ ).

For both the taxes and permits cases, we compute the dynamic programming equation and associated policy function in each period over a 20-year horizon by iterating backwards on the value function. A discrete grid size is chosen for the relevant state variable ( $p$  for the tax case and  $a$  for the permit case), and a hill-climbing algorithm is used to maximize the objective function, where we use a cubic spline interpolation to evaluate points between the discrete grid values. All calculations are performed in MATLAB.<sup>5</sup> The next two subsections are devoted to explaining briefly the numerical optimization procedure employed for each case (taxes and permits).

#### 4.2. Numerical optimization: taxes

Computing the quantities of interest in the tax and permits cases involves solving for the value function and associated policy function in every period using backwards induction. When the regulator uses taxes as her instrument, the period  $t$  state variable for each firm is the current emissions tax,  $p_t$ . The control variable for firm  $j$  ( $j \in \{h, l\}$ ) is the level of abatement in that period,  $q_t^j$ . Numerically iterating on the value function using backwards induction involves the following steps:

<sup>4</sup>Because  $\alpha$  is assumed to be constant from here onward, to avoid clutter we suppress it as an argument in the value functions.

<sup>5</sup>We computed results for the two-period problem analytically and, as a test of accuracy, compared them with those generated by the numerical optimization method. The results were within 0.01% in all cases.

1. In the final period,  $T$  (which equals 20 in this experiment), firm  $j$ 's dynamic programming equation is

$$V_T^j(p_T) = \max_{q_T^j} \left\{ -p_T(e^j - q_T^j) - (\theta + d)q_T^j - \frac{b^j(q_T^j)^2}{2} \right\} \quad (6)$$

because  $V_{T+1}^j(p_{T+1})$ , the future value function, equals zero. Procedurally, Eq. (6) is solved by partitioning the state space into a discrete grid and solving this problem for each value of  $p_T$  in the grid. This method gives the value function  $V_T^j(p_T)$  for period  $T$  evaluated at every point on the grid. It also produces the optimal policy function for firm  $j$ ,  $q_T^j(p_T)$ , which in turn yields the optimal abatement in period  $T$  for firm  $j$  given a tax of  $p_T$ .

2. Stepping back one period, firm  $j$  now solves the following problem:

$$V_{T-1}^j(p_{T-1}) = \max_{q_{T-1}^j} \left\{ -p_{T-1}(e^j - q_{T-1}^j) - (\theta + d)q_{T-1}^j - \frac{b^j(q_{T-1}^j)^2}{2} + \delta V_T^j(p_T) \right\},$$

where each firm is assumed to know the continuation value,  $V_T^j(p_T)$ . Firm  $j$  must now consider two quantities in its choice of abatement: (1) the current payoff, and (2) the effect that its choice of abatement this period will have on the tax next period. In this model we assume the regulator believes firms are acting non-strategically. Specifically, firm  $j$  knows that the regulator will set  $p_T$  where  $MB = E[MC]$ . That is, in any period  $t$ , the regulator's rule for setting the future tax is  $p_{t+1} = (v(p_t - Bq_t) - Bw)/(v - B)$ . Both firms know this and, given abatement by the other firm ( $q_{T-1}^j$ ), firm  $j$  can solve its problem. The Nash equilibrium of this game yields the abatement levels for both firms given  $p_{T-1}$ . Therefore, for period  $T - 1$  we have the policy functions  $q_{T-1}^j(p_{T-1})$  and the value functions  $V_{T-1}^j(p_{T-1})$ .

3. Knowing  $V_{T-1}^j(p_{T-1})$  for both firms, we repeat the preceding step to compute  $q_{T-2}^j(p_{T-2})$  and  $V_{T-2}^j(p_{T-2})$ . This backwards induction procedure is continued back to period 1. Through this exercise, we have discovered how each firm optimally responds, in any period  $t$ , to the emissions tax set by the regulator in that period,  $p_t$ .
4. To determine welfare, we need a value of the emissions tax in the first period,  $p_1$ , which is determined by the regulator as if  $\theta = 0$ . For each set of parameter values, this tax is calculated and the taxes and emissions are simulated over the 20-year horizon. Welfare is calculated as the net present value of the stream of total benefits less aggregate total cost.

#### 4.3. Numerical optimization: tradeable permits

The permit case is very similar to the tax case, with one simplification: computing the Nash equilibrium is not required, because the follower firm ( $j = F$ ) takes the permit price set by the leader firm ( $j = L$ ) as given and does not attempt to manipulate future behavior of the regulator. When the regulator uses permits as the instrument to reduce pollution, the leader's period  $t$  state variable is the total allocation of permits,  $a_t$ , and the leader's control variable is the permit price,  $p_t$ . Again, we will solve this dynamic optimization problem by value function iteration starting at period  $T$ . The procedure is as follows:

1. As in the tax case, the state space is discretized into a grid and cubic spline interpolation is used to evaluate points between the discrete grid values of  $a$ . In period  $T$ , the follower solves:

$$\max_{q_T^F} \left\{ p_T(a_T^F + q_T^F - e^F) - q_T^F(d + \theta) - \frac{b^F(q_T^F)^2}{2} \right\}$$

which implies period  $T$  abatement for the follower firm of  $q_T^F(p_T) = (p_T - (d + \theta))/b^F$ . The leader takes this information, along with the market-clearing condition, into account when setting the period  $T$  price. The market-clearing condition gives the leader's abatement as a function of its own choice of the permit price:

$$q_T^L(p_T) = e^F + e^L - a_T^F - a_T^L - q_T^F(p_T).$$

The leader chooses  $p_T$  in the period  $T$  dynamic programming equation:

$$V_T^L(a_T) = \max_{p_T} \left\{ p_T(a_T^L + q_T^L(p_T) - e^L) - (d + \theta)q_T^L - \frac{b^L(q_T^L)^2}{2} \right\}.$$

This calculation produces the optimal period  $T$  policy function for the leader,  $p_T(a_T)$ , and the period  $T$  value function for the leader, to be used in the calculation for period  $T - 1$ .

2. Stepping back one period, the future value function is known. Furthermore, the follower still acts non-strategically, and therefore has the same policy response as in period  $T$ ,  $q_{T-1}^F(p_{T-1}) = (p_{T-1} - (d + \theta))/b^F$ . The leader solves the following problem:

$$\begin{aligned} &V_{T-1}^L(a_{T-1}) \\ &= \max_{p_{T-1}} \left\{ p_{T-1}(a_{T-1}^L + q_{T-1}^L(p_{T-1}) - e^L) - (d + \theta)q_{T-1}^L - \frac{b^L(q_{T-1}^L)^2}{2} + \delta V_T^L(a_T) \right\}. \end{aligned}$$

The leader must now consider not only its current-period payoff, but the effect that a current choice  $p_{T-1}$  will have on the future allocation of permits,  $a_T$ . The leader also knows that the regulator acts non-strategically in this fashion. That is, the regulator sets the new allocation of permits ( $a_T$  in this case) where marginal benefit equals expected marginal cost. In particular, the regulator's updating rule is as follows:

$$a_{t+1}(a_t, p_t) = \frac{p_t + Ba_t - w - ve}{B - v}.$$

The leader's knowledge of the regulator's updating rule is very important, as this firm now knows exactly how its current choice of permit price ( $p_{T-1}$ ) affects the continuation value of the program. The firm solves the dynamic programming equation above, and obtains the policy function  $p_{T-1}(a_{T-1})$  and the resultant value function  $V_{T-1}^L(a_{T-1})$ .

3. This procedure is continued backwards to the first period. The initial allocation of permits,  $a_1$ , is chosen by the regulator as if  $\theta = 0$ , and the permit market is simulated forward 20 periods.

#### 4.4. Results of numerical optimization

Figs. 2 and 3 illustrate the results for the 20-period dynamic programming problem under both taxes and permits assuming the following parameter values:  $v = -1$ ,  $e^j = e^{-j} = 25$  ( $e = 50$ ),

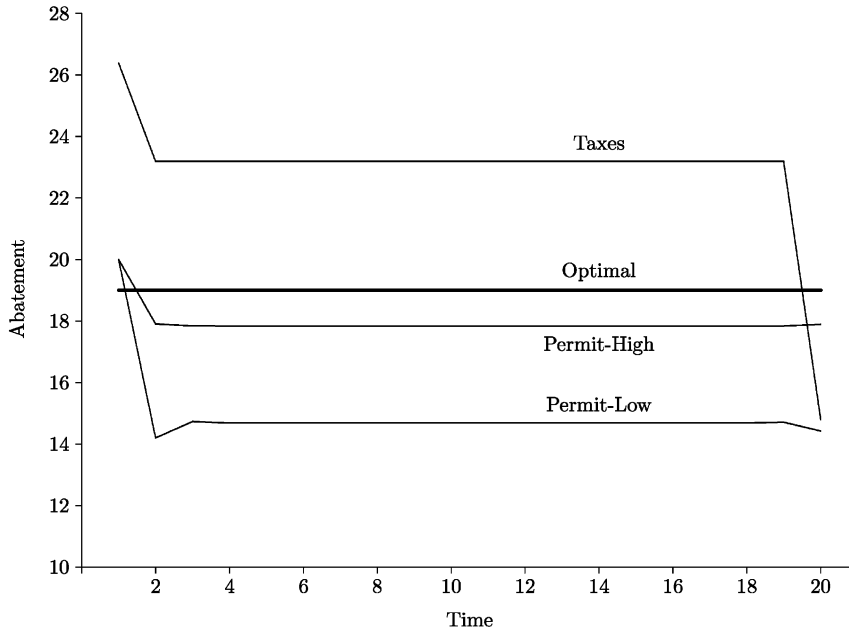


Fig. 2. Abatement levels, baseline case ( $v = -1$ ,  $e = 50$ ,  $\delta = 1$ , and  $\theta = 2$ ).

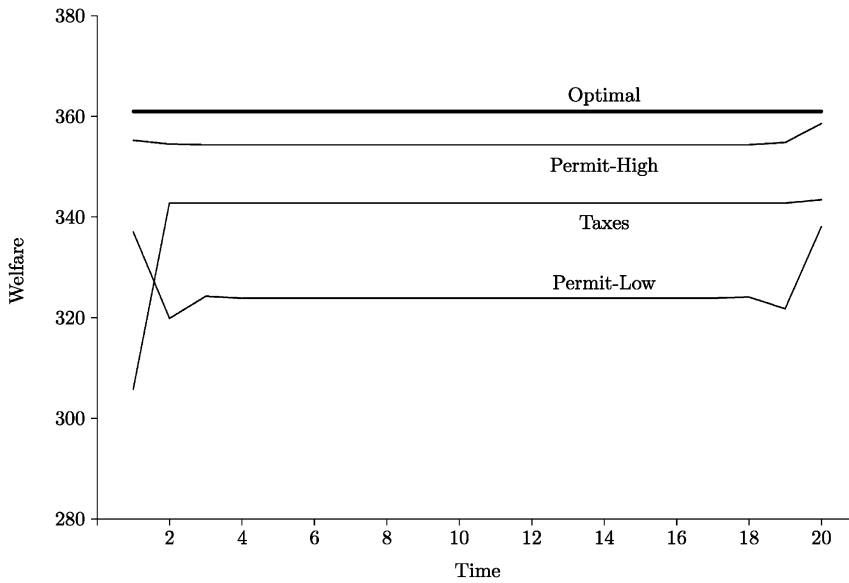


Fig. 3. Welfare levels, baseline case ( $v = -1$ ,  $e = 50$ ,  $\delta = 1$ , and  $\theta = 2$ ).

$\delta = 1$  and  $\theta = 2$ . Fig. 2 shows abatement quantity in each time period for four cases: (a) taxes, (b) permits when the low-cost firm sets permit price ( $b^L = 1.5$ ,  $b^F = 3$ ), (c) permits when the high-cost firm sets permit price ( $b^L = 3$ ,  $b^F = 1.5$ ), and (d) the optimal solution. In the optimal solution, abatement is such that the marginal benefit and the marginal cost of abatement are

equal:  $w + vq = (\theta + d) + Bq$ . With these parameter values, the optimal solution involves setting abatement in each period equal to 19.

In the tax case, abatement in period 1 is equal to 26.31, falls to 23.19 in period 2, stays there until the final period, then drops to 14.81. With the exception of the final period, firms abate more than in the optimal solution. Overabatement is caused by firms choosing to increase abatement to obtain low taxes in the next period. In the final period, there is no incentive to overabate, and firms in fact underabate relative to what is optimal.

With permits, the regulator allocates 30 permits in the initial period. This fixes first-period abatement at 20. After the first period, abatement quantity differs depending upon whether the low-cost firm or the high-cost firm sets the permit price. When the low-cost firm sets permit price, abatement quantity settles to a value of 14.69 after a few periods. When the high-cost firm sets the permit price, abatement quantity settles to 17.84 after a few periods. With permits, firms have a strategic incentive to set high permit prices to obtain more permits in the next period. Monopoly pricing reinforces the strategic incentive to set high prices when the low-cost firm sets prices, which results in a large allotment of permits and low abatement. When the high-cost firm is the leader, the strategic impulse to set high prices is offset somewhat. This results in lower permit prices, fewer permits allocated in subsequent periods, and more abatement, than in the case in which the low-cost firm sets prices. Even when the high-cost firm sets prices, however, abatement is lower than the optimal level.

Fig. 3 shows welfare in each period for each of the four cases (optimal solution, taxes, permits with the low-cost firm setting the price, and permits with the high-cost firm setting the price). With the parameter values assumed in this example, welfare is highest in the case with permits where the high-cost firm sets price. The deadweight loss in this case is quite small. The present value of welfare in the optimal solution over the entire 20-year horizon is 7220. For the case with permits where the high-cost firm sets price, the present value of welfare is 7092. The deadweight loss for this case is only 1.8%. The present value of welfare under emissions taxes is 6818 (deadweight loss of 5.6%). Welfare is lowest under permits where the low-cost firm set prices. The present value of welfare in this case is 6499 (deadweight loss of 10.7%). Regarding welfare generated per period, the order of the cases is consistent for all periods except the first. In periods 2–20, welfare is highest under permits when the high-cost firm sets permit price, followed next by taxes, and then by permits when the low-cost firm sets permit price.

In the first period, however, welfare is higher under permits than under taxes regardless of which firm sets the permit price. Firms cannot manipulate the number of permits allocated in the first period, which limits first-period deadweight loss. Almost all of the deadweight loss in the first period is due to different marginal abatement costs for the two firms. In contrast, under emissions taxes firms set first-period abatement higher than in any other period. Firms do so to decrease the regulator's belief about  $\theta$ , which results in the regulator choosing a low tax in the next period. Once the regulator believes that  $\theta$  is low, the firms do not need to abate as much in order to maintain that belief. If we had chosen a time horizon short enough to give the first period a dominant role, welfare would have been higher under permits than under taxes regardless of which firm had set the permit price.

For the parameter values chosen in this example, firms overabate in the steady state when taxes are used (consistent with Proposition 4), and underabate when permits are used. Perhaps a more striking result is that steady-state welfare comparisons appear to hinge on whether the low- or the

Table 1  
Steady-state abatement levels

Case	Parameters			Optimal solution	Taxes	Permits (low-cost leader)	Permits (high-cost leader)
	$v$	$e$	$\theta$				
1	-0.25	200	2.0	30.4	44.0	19.5	25.9
2	-0.50	100	2.0	25.3	33.8	17.7	22.6
3	-1.0	50	2.0	19.0	23.2	14.7	17.8
4	-2.0	25	2.0	12.7	14.3	10.8	12.3
5	-3.0	13.3	2.0	9.5	10.3	8.5	9.4
6	-0.25	200	-2.0	33.6	46.9	22.8	29.2
7	-0.50	100	-2.0	28.0	36.2	20.5	25.5
8	-1.0	50	-2.0	21.0	24.9	16.8	20.0
9	-2.0	25	-2.0	14.0	15.4	12.2	13.7
10	-3.0	13.3	-2.0	10.5	11.2	9.5	10.4

high-cost firm sets the price in the permit market. To explore whether this result is a peculiarity of these specific parameter values or is more robust, we compare steady-state abatement and welfare for an infinite-horizon problem over a range of parameter values. In all, 10 cases are explored under different values of the intercept term for the marginal abatement cost ( $\theta$ ), the slope of the marginal benefit of abatement ( $v$ ), and the total unregulated emissions ( $e$ ).

Table 1 provides a description of each of the 10 cases and gives the steady-state abatement levels for the optimal solution, the case with taxes, and the two cases with permits.<sup>6</sup> Consistent with Proposition 4, in all cases using taxes as the instrument leads to overabatement in the steady state. As noted in Section 3.2, the determination of whether firms overabate or underabate when permits are used depends on the magnitude of the strategic term. Typically, as suggested earlier, the value of a firm's optimal program will be increased by an increase in the total allocation of permits, meaning that price is set above marginal abatement cost, and too little abatement will occur. Although the opposite result can be achieved in theory, for all parameter values explored in this exercise firms underabate when strategically interacting in a permit market.

Table 2 compares steady-state welfare levels for the various regulatory regimes. Following Weitzman [12] one might expect that the relative slopes of the marginal benefit and marginal cost functions determine whether taxes or permits yield the higher welfare level. This is not the case for the examples explored here. Consistent with the example in Fig. 3, we find that the dominance of taxes over permits depends entirely on the abatement costs of the leader and follower firms in the permit market. For all 10 cases, when the leader has high costs (relative to the follower), permits dominate taxes in terms of welfare; the opposite is true with a low-cost leader.

## 5. Conclusions

When an environmental regulator does not have complete information about firms' abatement costs, the firms can use this informational asymmetry to their advantage. Firms know that their

<sup>6</sup>Note that the case illustrated in Figs. 2 and 3 corresponds to case 3 in the table.

Table 2  
Steady-state welfare levels

Case	Parameters			Optimal solution	Taxes	Permits (low-cost leader)	Permits (high-cost leader)
	$v$	$e$	$\theta$				
1	-0.25	200	2.0	577.6	461.6	455.9	533.0
2	-0.50	100	2.0	481.3	426.4	404.1	459.7
3	-1.0	50	2.0	361.0	342.7	323.9	354.3
4	-2.0	25	2.0	240.7	236.3	227.5	239.4
5	-3.0	13.3	2.0	180.5	178.8	174.1	180.1
6	-0.25	200	-2.0	705.6	594.4	586.7	663.4
7	-0.50	100	-2.0	588.0	537.3	514.0	568.6
8	-1.0	50	-2.0	441.0	425.1	406.5	435.7
9	-2.0	25	-2.0	294.0	290.4	282.2	293.2
10	-3.0	13.3	-2.0	220.5	219.1	214.9	220.3

behavior in a given time period is a signal of costs to the regulator, and that she takes this behavior into account when making policies in subsequent periods. We have explored the effects of this strategic behavior on the price–quantity comparison.

With an emissions tax, firms have a strategic incentive to overabate in order to obtain a low tax in the next period. With permits, firms have a strategic incentive to set the permit price high in order to obtain a large number of permits in the next period, leading to underabatement. Though a permit buyer has an incentive to lower the price in order to make purchasing less expensive, in all of the cases we examined the strategic effect dominated and permits prices were higher than marginal abatement costs. In the steady-state equilibrium, taxes led to overabatement while standards led to underabatement.

We used numerical optimization to compare prices and quantities and, in the spirit of Weitzman [12], showed whether the comparison depends on the relative slopes of marginal cost and marginal benefit. In our model, which policy instrument is least distorting depends upon which firm sets the permit price. When the low-cost firm sets the price, taxes outperform permits in terms of welfare. On the other hand, when the high-cost firm sets the price, permits outperform taxes. For the parameters used, when taxes dominate they do so by a small margin. When permits dominate, they can dominate by a much larger margin (up to 15%).

A number of natural extensions of this work could be considered. One could consider the case in which the regulator is sophisticated enough to know that she may be misled by the firms. In this case, both the regulator and the firms are strategic, which generates a complicated dynamic game of incomplete information. Because both the firms and the regulator would behave strategically in the resulting asymmetric-information game, a model of this sort would present a more difficult challenge technically. A second extension would be to allow firms to bank permits from one period to the next, as they can in the US sulfur dioxide allowance-trading program. It appears that firms would not wish to bank in our model. Doing so would tend to make the regulator believe that marginal costs were lower so that she would allocate fewer permits. In addition, with banking the regulator would allocate fewer permits for any given estimate of costs because she would know

that some first-period permits are now available in the second period. The question of equilibrium with banking, and possibly borrowing, deserves further study.

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